Mathematics for Amateur Radio

4

Sooner or later, most hams will find they need to make some sort of measurement or perform a calculation as part of their hobby. This is true whether they are calculating an antenna length or designing a new piece of station equipment. When they do, they will be using mathematics. The math skills required for most electronics calculations can be developed and used by just about anyone.

This chapter, written by Larry Wolfgang, WR1B, provides a brief review of the most important math concepts needed for electronics and Amateur-Radio-related use. It will serve as a refresher for those hams who may have been familiar with the topics, but who have long since forgotten how to apply them. The examples will also help those who have no prior math background to work through many of the calculations associated with this *Handbook*. Those readers who would like a more detailed explanation should turn to the Math Unit of ARRL's *Understanding Basic Electronics*.

Software to perform calculations is discussed in ARRL's *Personal Computers in the Ham Shack*, and new packages are reviewed from time to time in *QST*.

Mathematical Terms and Symbols

Mathematics uses letters, symbols and odd-looking characters to represent various quantities in a kind of short-hand notation we call equations. To those unfamiliar with the language of mathematics, these strange names and symbols can be very confusing. Once you have learned some basic terms and understand what the symbols represent, the elegance of an equation can begin to come through. In this section we will introduce some of the most common mathematical terms and symbols.

DEFINITIONS OF MATHEMATICAL TERMS

Algebra—The branch of mathematics that uses letter symbols to represent various quantities, and which establishes rules for manipulating these expressions. Much of the discussion in this chapter involves the rules of algebra.

Binary number system—A number system that uses only two symbols, 0 and 1. The binary system is very useful in digital electronics, because most digital electronics circuits only have to measure two voltage or current conditions: ***on*** or ***off***. Most of these circuits represent the ***on*** condition as a 1 and the ***off*** condition as a 0. (See also ***Decimal number system***, ***Hexadecimal number system*** and ***Octal number system***.)

Cross multiplication—The most common equation-solving technique used with proportions. This involves moving terms diagonally across the equal sign. We can use the letters a, b, c and d to represent four terms of a proportion:

$$\frac{a}{b} = \frac{c}{d}$$

Then by cross multiplication, we can also write the following equivalent expressions:

$$ad = bc \text{ and } \frac{a}{c} = \frac{b}{d}$$

Cube—Multiplying a number or quantity by itself three times. Cubing a quantity means it is raised to the third power, or has an ***exponent*** of 3. ($2^3 = 2 \times 2 \times 2 = 8$)

Cube root—That value which, when multiplied by itself three times, gives the value whose cube root you want to find. ($\sqrt[3]{8} = 8^{1/3} = 2$ and $\sqrt[3]{-8} = -8^{1/3} = -2$) Odd powered roots have only one possible value.

Decimal number system—A number system that uses ten symbols, 0 through 9, to count, measure and calculate. The most common number system. (See also ***Binary number system***, ***Hexadecimal number system*** and ***Octal number system***.)

Equation—A statement of mathematical balance. All that appears on one side of the equal sign (=) is equivalent to any expression on the other side. The two sides usually don't appear identical (2 = 2), but the expression on one side represents the expression on the other side ($x = 2$). The x here represents a ***variable***, or unknown quantity.

Exponent—A value following a number, raised above the line of the number, or written as a ***superscript***, to show the number is to be multiplied by itself. (10^3 indicates that 10 is to be multiplied by itself three times — $10 \times 10 \times 10$.) The rules of working with exponents are covered later in this chapter.

Formula—Another name for an ***equation***, especially when it represents a procedure used to calculate some quantity. ($E = I R$ is a formula that tells us to multiply current times resistance to find voltage.)

Hexadecimal number system—A number system that has 16 characters; labeled 0 through 9, A, B, C, D, E and F. The hexadecimal system (often abbreviated *hex*) is convenient for use with digital computers because hexadecimal digits can be coded as groups of four binary digits. In this case, 0001 represents hex 1, 1000 represents hex 8, 1010 represents hex A and 1111 represents hex F. (See also ***Binary number system***, ***Decimal number system*** and ***Octal number system***.)

Infinity—The term used to describe the mathematical concept of having no boundaries. There is no "largest number" or "smallest number," because you can always add 1 to obtain a larger number, or further divide to obtain a smaller one.

Integers—The "counting numbers," such as 1, 2, 3, 4, 5. Integers also include negative values. The number line of **Fig 4.1** is helpful to picture positive and negative integers. (See also ***Real numbers***.)

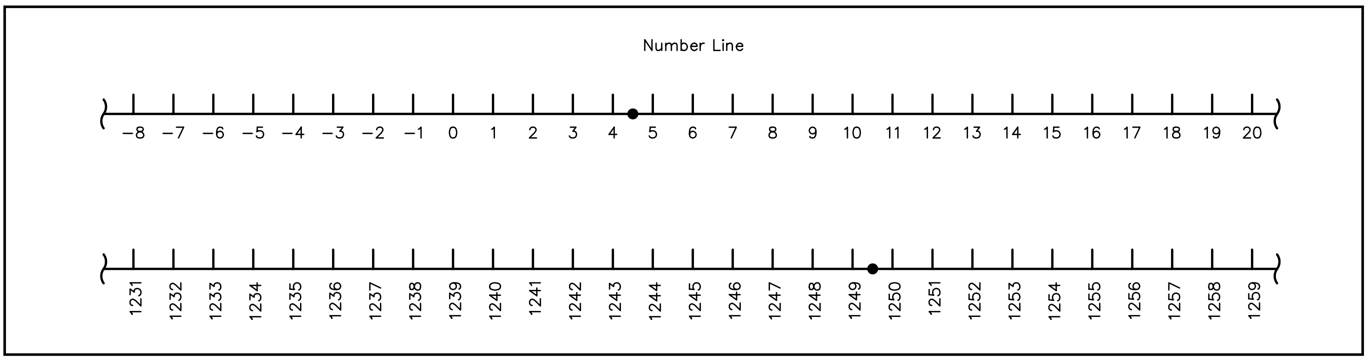

Fig 4.1 — The number line gives us a way to represent all numbers, both positive and negative, and is useful for remembering arithmetic operations.

Octal number system—A number system that uses eight characters, 0 through 7. This system is often used with digital computers, because groups of three binary digits can be coded to represent an octal digit. For example, 001 represents octal 1, 010 is the same as octal 2 and 111 represents octal 7. (See also ***Binary number system***, ***Decimal number system*** and ***Hexadecimal number system***.)

Power of 10—The exponent used with 10 when a number is written in exponential or scientific notation. The exponent tells how many places and in what direction the decimal point is moved.

Proportion—Two ratios that are equal to each other (or both equal to the same quantity). Proportions are a powerful mathematical tool because you can often write an equation to calculate some unknown quantity based on your knowledge of another ratio. Proportions are useful because when you know three of the four quantities, it is a simple matter to find the fourth. Later in this chapter we show you how to use proportions to convert between US Customary and metric system measurements.

Radical sign—$\sqrt{\ }$. A symbol written with a number or mathematical expression under the line, to represent a square root, such as $\sqrt{4} = 2$. If there is a superscript number in front of the radical sign, then it represents the root indicated.

$$\sqrt[3]{8} = 2$$

Ratio—A fraction, with one quantity divided by another. The value of π is the ratio of the circumference of a circle (C) to the diameter of the circle (d), for example.

$$\left(\pi = \frac{C}{d}\right)$$

Voltage standing-wave ratio (VSWR or SWR) is the ratio of maximum voltage on a feed line to the minimum voltage on the feed line. Written as a fraction, we use this ratio to form an equation that shows one way to calculate SWR:

$$SWR = \frac{V_{max}}{V_{min}}$$

Real numbers—All possible numbers, including all the fractions between integers. (Fractions can be written as a ratio of two numbers, or as a decimal value that is the result of the division. 4.5 and $4^1/_2$ represent the same real number. The decimal value is often only an approximation, however, such as 6.333 and $6^1/_3$.)

Reciprocal—A quantity divided into 1 (often written as 1/*x*). Reciprocals are so important that the quantity is often given a name of its own. For example, in electronics, the reciprocal of resistance is called conductance. Using letter symbols to represent the quantities (R is resistance and G is conductance).

$$\left(\frac{1}{R} = G \text{ and } \frac{1}{G} = R\right)$$

Root (of a number)—A value which, when multiplied by itself the specified number of times, gives the value whose root you want to find. Most common in electronics is the ***square root*** of a number, and occasionally the ***cube root***. Roots may be written with a ***radical*** sign ($\sqrt{\ }$) or as a fractional ***exponent***. ($\sqrt{4} = 4^1/_2 = 2$)

Square—Multiplying a number or quantity by itself. Squaring a quantity means it is raised to the second power, or has an ***exponent*** of 2. ($2^2 = 2 \times 2 = 4$)

Square root—That value which, when multiplied by itself, gives the value whose square root you want to find. Actually, there are two values for a square root. ($\sqrt{4} = 4^1/_2 = +2$ and –2) Even-powered roots have both the positive and negative values possible.

Subscript—A number or expression following a variable, written slightly lower than the line of the variable; R_1, R_2, E_3 and E_4 are examples of quantities with subscripts to distinguish similar, but different quantities.

Superscript—A number or expression written following a number or variable, written slightly higher than the line of the number or expression; 5^2, x^3, $(25 + I)^2$ and E^2 are examples of expressions with superscripts.

Variable—An expression that can take on different values. Variables are sometimes given ***subscripts***.

GREEK ALPHABET

Upper- and lower-case characters of the Greek alphabet are often used to represent various measurements and constant values. Few English-speaking people are familiar with Greek, so some of these characters can look pretty strange. **Table 4.1** shows the upper and lower case Greek alphabet, the character pronunciations, and the electrical and electronics quantities some of these characters often represent.

Table 4.1
The Greek Alphabet and Common Electronics Quantities

Greek letter	*Pronunciation*	*Upper Case*	*Common Use*	*Lower Case*	*Common Use*
Alpha	'al-fə	Α	Angle of a triangle	α	Transistor common-base current gain
Beta	'bāt-ə	Β	Angle of a triangle	β	Transistor common-emitter current gain
Gamma	'gam-ə	Γ	Transmission line voltage reflection coefficient	γ	Phase
Delta	'del-tə	Δ	Change in quantity	δ	
Epsilon	'ep-sə-län	Ε		ε	Dielectric constant, permittivity
Zeta	'zāt-ə	Ζ		ζ	
Eta	'āt-ə	Η		η	
Theta	'thāt-ə	Θ	Angles	θ	Angles
Iota	i-'ot-ə	Ι		ι	
Kappa	'kap-ə	Κ		κ	
Lambda	'lam-də	Λ		λ	Wavelength
Mu	myü	Μ		μ	Metric prefix for 10^{-6}, permeability
Nu	nü	Ν		ν	
Xi	ksi	Ξ		ξ	
Omicron	'äm-ə-krän	Ο		ο	
Pi	pī	Π		π	3.14159 (ratio of circumference to diameter of a circle)
Rho	rō	Ρ		ρ	Transmission line reflection coefficient, resistivity
Sigma	'sig-mə	Σ	Summation of a series	σ	
Tau	tau	Τ		τ	Time constant, LC circuits
Upsilon	'yüp-sə-län	Υ		υ	
Phi	fi	Φ	Angles	φ	Angles
Chi	ki	Χ		χ	
Psi	si	Ψ		ψ	
Omega	o-'meg-ə	Ω	Ohm, resistance, normalized frequency	ω	Frequency in radians per second (2πf), angular velocity

TABLE OF MATHEMATICAL SYMBOLS

In addition to Greek characters and other letter symbols, there are many special math symbols used when we write equations. **Table 4.2** shows many of these common math symbols.

Table 4.2
Some Common Mathematical Symbols

Symbol	*Meaning*
+	Addition, plus
–	Subtraction, minus
±	Plus or minus
×, •,*	Multiplication, multiply by
÷, /	Division, divide by
=	Equal to
≠	Not equal to
≈	Approximately equal to
~	Similar, equivalent
<	Less than
≤	Less than or equal to
>	Greater than
≥	Greater than or equal to
:	Ratio of, is to
∝	Proportional, varies directly as
∴	Therefore
°	Degree
∠	Angle
∟	Right angle
⊥	Perpendicular to
‖	Parallel to
≡	Identical to
∞	Infinity
$\sqrt{\ }$	Radical, square root (Also written with a superscript before the symbol to express other roots, such as $\sqrt[3]{\ }$ to represent a cube root.)
∫	Integration
Σ	Summation

Significant Figures and Decimal Places

Any measurement is only as good as the measuring instrument used, and as reliable as the person making the measurement. The *accuracy* of a measurement refers to how close the value is to an accepted value or standard. You might use a dip oscillator to measure the resonant frequency of a radio circuit. The measured value will probably not be very accurate, since dip oscillators are usually designed only to measure a general range. If you want to ensure that your transmitter is operating inside the amateur band edge, you might try a frequency counter or crystal calibrator, for example. So the measuring instrument plays the greatest role in determining the accuracy of a measurement. (This assumes the person taking the measurement understands how to use the instrument to take full advantage of it. An operator who does not know how to use or read the instrument properly will not obtain accurate measurements!)

Precision refers to the repeatability of a measurement. You might take five frequency readings using the dip meter mentioned above, with all five readings being 7.14 MHz. This set of measurements would have good *precision*, but because the instrument is not designed for high *accuracy*, you can't be sure of the actual frequency. Other factors might affect the precision of a measurement, such as the operator's skill at adjusting the measuring instrument, errors in reading the scale and the operation of the instrument itself.

The value you can read from the scale of a measuring instrument helps determine its precision. If you are using a ruler marked off only to the nearest quarter inch, you may be able to estimate measurements to an eighth inch, but you certainly can't read that scale to the nearest thirty-second of an inch! It will be difficult to measure two objects that differ in length by a sixteenth of an inch with this ruler. Precision also indicates the *resolution* of a measurement, or how small a change can really be detected.

The *significant figures* of a measurement represent all the digits that you can read directly from the scale, plus one digit that is estimated. **Fig 4.2** shows a voltmeter scale marked from 0 to 10 V, with lines indicating every 0.2 V. You can see that the needle indicates a value between 4.6 and 4.8 V, and perhaps you can even tell if the needle is more or less than half way between the marks. You know the reading is a little less than 4.7 V, but you really can't be sure how much less. You can estimate that the reading is 4.68 perhaps, but the 8 can't be read directly from the scale. Someone else might look at the same reading on the same meter and estimate the value at 4.67 V or even 4.69 V. None of these readings is more correct than any other, because each represents an *estimate* of the value of the last digit. This reading has three significant figures.

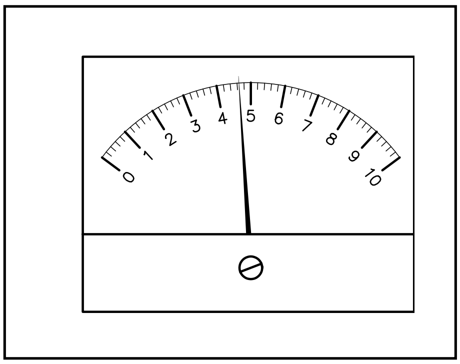

Fig 4.2 — This voltmeter scale reads from 0 to 10 V, with marks every 0.2 V. The meter is reading a value greater than 4.6 but less than 4.7 V. With care you may be able to estimate the reading as 4.68 V, but the digit 8 really only represents a guess. That digit is uncertain because you cannot read it directly from the scale.

Any calculations made involving this measurement are limited by the accuracy of the reading. It would be completely unreasonable to say that 4.68 V produces a current of 17.01818 mA when connected to a 275-Ω resistor, even though your calculator shows all these digits.

The rules of significant figures tell us how many digits to include in any calculation based on a measured quantity. They also help us predict the accuracy of a calculation based on real component tolerances. These rules allow us to specify the accuracy of the calculated value, as it relates to the measurements on which the calculation is based. There are six rules to tell you how to count and write significant figures in a measured or calculated quantity.

1. All nonzero digits are significant: 275.4 mA has four significant digits.

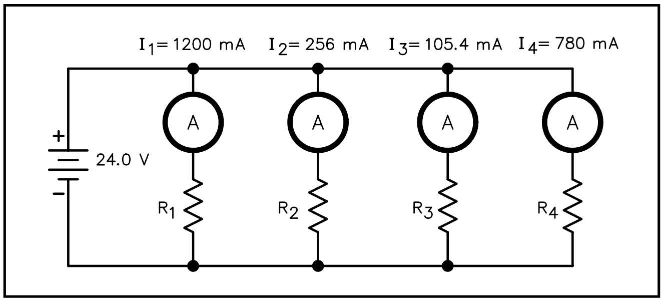

Fig 4.3 — This parallel circuit has four branches connected to the battery. Ammeters measure the current through the branches, with the readings indicated. This circuit illustrates measurements with two, three and four significant figures (I_1 and I_4, I_2, I_3 respectively).

```
 ↓ ————————— Left most uncertain value
               in measurements
  1200   mA
   256   mA
   105.4 mA
+  780   mA
———————————
 ↓ ————————— Right most significant
  2341.4       figure in answer
  2300 ← Total current value rounded
         off to show the hundreds column
         as the last significant figure
```

Fig 4.4 — This calculation shows the proper use of the rule for addition and subtraction using significant figures. The current measurements from Fig 4.3 are added to calculate the total circuit current. The resulting value is then rounded off to make the hundreds column the last significant figure.

2. All zeros between nonzero digits are significant: 25.004 m has five significant digits.
3. Zeros to the right of a nonzero digit, but to the left of an understood decimal point are not significant unless they are specifically indicated to be significant. You can indicate such zeros to be significant by drawing a bar over the rightmost significant zero: 21100000 hertz has three significant figures; $2110\bar{0}000$ hertz has five significant figures.
4. Zeros to the right of a nonzero digit, but to the left of an expressed decimal point are significant: 21100. kHz has five significant figures.
5. Zeros to the right of a decimal point but to the left of all nonzero digits are not significant: 0.001702 A has four significant figures. (There is a zero before the decimal point to indicate that no digits to the left of the decimal point were dropped. Also notice that the zero between the 7 and the 2 is significant — remember rule 2.)
6. All zeros to the right of a decimal point and following a nonzero digit are significant: 2.00 V has three significant figures.

There are a few rules for determining the number of significant figures that result when you use measured values in a calculation. These rules are important to ensure that you don't imply a result to a greater precision than the measurements would allow.

Notice we said *measured* values here. Never use the number of digits in the value of a physical constant to limit the number of significant figures in a calculation. For example, the constant 2 or the value of π won't limit the number of significant figures in a calculation of reactance. ($X_L = 2 \pi f L$) Likewise, values of trigonometric functions and logarithm values aren't usually limited by the number of significant figures in the number used to find the function value.

When adding or subtracting measurements, remember that the rightmost significant figure represents an uncertain value. The rightmost significant figure in a sum or difference calculation occurs in the leftmost place that an uncertain value occurs in any of the measured quantities. The following example illustrates this rule.

Fig 4.3 shows a parallel circuit with four branches. Ammeters measure the current through each branch. The four current measurements are 1200 mA, 256 mA, 105.4 mA and 780 mA. What is the total current supplied to this circuit? The first measurement has only two significant figures, and the hundreds column represents an uncertain value. The second measurement has three significant figures and the units column is uncertain. The third current has four significant figures, with the tenths column being uncertain. The last current value has two significant figures, and the tens column is uncertain. **Fig 4.4** shows how to determine the last significant place in the answer. Round off the result of this addition to the hundreds place, so this circuit has a total current of 2300 mA. (The rules for rounding numbers are covered later in this section.)

When you multiply or divide measured quantities, the answer cannot have more significant figures than the least precise factor. As an example, use Ohm's Law to calculate the resistor values in Fig 4.3. **Fig 4.5** shows how to determine the significant figures for these calculations.

ROUNDING VALUES

After you determine which digit is the last significant figure in a calculation, you will have to round off the arithmetic answer. Four rules govern how to round off values properly.

1. If the first digit to be dropped is 4 or less, the preceding digit is not changed: 456351 rounded to three significant figures becomes 456000 (with no decimal point at the end).

2. If the first digit to be dropped is 6 or more, the preceding digit is increased by 1: 456351 rounded to two significant figures becomes 460000 (with no decimal point at the end).

3. If the digits to be dropped are 5 followed by digits other than zeros, the preceding digit is increased by 1: 456351 rounded to four significant figures becomes 456400 (with no decimal point at the end).

4. If the digits to be dropped are 5 followed by zeros (the digit to be dropped is exactly 5) the preceding digit is *not* changed if it is even; it is raised by 1 if it is odd: 456350 rounded to four significant figures becomes 456400 but 456450 rounded to four significant figures also becomes 456400 (with no decimal point at the end). (Another way to think of this rounding rule is that when the digit to be dropped is exactly 5, we round to the *even* value.)

$$R_1 = \frac{E}{I_1} = \frac{\overbrace{24.0}^{\text{Three Significant Figures}}\ \text{V}}{\underbrace{12}_{\text{Two Significant Figures}}00\ \text{mA}} = \frac{24.0\ \text{V}}{1.2\ \text{A}} = \underbrace{20.}_{\text{Two Significant Figures}}\ \Omega$$

$$R_2 = \frac{E}{I_2} = \frac{\overbrace{24.0}^{\text{Three Significant Figures}}\ \text{V}}{\underbrace{256}_{\text{Three Significant Figures}}\ \text{mA}} = \frac{24.0\ \text{V}}{0.256\ \text{A}} = 93.75\ \Omega = \underbrace{93.8}_{\text{Three Significant Figures}}\ \Omega$$

$$R_3 = \frac{E}{I_3} = \frac{\overbrace{24.0}^{\text{Three Significant Figures}}\ \text{V}}{\underbrace{105.4}_{\text{Four Significant Figures}}\ \text{mA}} = \frac{24.0\ \text{V}}{0.1054\ \text{A}} = 227.70398\ \Omega = \underbrace{228}_{\text{Three Significant Figures}}\ \Omega$$

$$R_4 = \frac{E}{I_3} = \frac{\overbrace{24.0}^{\text{Three Significant Figures}}\ \text{V}}{\underbrace{78}_{\text{Two Significant Figures}}0\ \text{mA}} = \frac{24.0\ \text{V}}{0.78\ \text{A}} = 30.76923\ \Omega = \underbrace{31}_{\text{Two Significant Figures}}\ \Omega$$

Fig 4.5 — These calculations show the proper use of the rule for multiplication and division using significant figures. The battery voltage and current measurements from Fig 4.3 are used to calculate the four resistor values. The resulting values are rounded off to show the proper number of significant figures.

Laws of Exponents

Exponents tell how many times a number or quantity is to be multiplied by itself. Equations often involve terms that include exponents. Two special cases with exponents are worth mention before we cover the rules for mathematical operations with exponents:

$a^1 = a$ and $a^0 = 1$

Any value raised to a power of 1 gives the value itself, and any value raised to the zero power is 1. We can also use numbers to give a few examples:

$10^1 = 10$, $5^1 = 5$ and $3^1 = 3$

$10^0 = 1$, $5^0 = 1$ and $3^0 = 1$

When you know the basic rules of algebra involving exponents, you will be able to manipulate the terms in an equation. There are only a few rules to remember, and they involve multiplication and division of numbers with exponents.

1. If you are adding, subtracting, multiplying or dividing two numbers involving exponents, calculate the values indicated by the exponents first, then perform the indicated operation on the numbers:
   $a^x \times b^y$ can't be simplified unless you know the values of the variables.
   $2^3 \times 4^2 = 8 \times 16 = 128$
2. If the multiplication involves a variable raised to different exponents, you can add the exponents:
   $a^x \bullet a^y = a^{x+y}$
   $2^3 \bullet 2^4 = 2^{3+4} = 2^7 = 8 \bullet 16 = 128$
   Notice the "multiplication dot" used in this example. It is also common practice to omit any symbol when multiplication of variables is intended, and there is no chance of confusion, such as would occur if you were writing two numbers:
   We can write this as $a^x\, a^y = a^{x+y}$ but we would not write 8 16 to indicate 8 • 16.
3. For division of a variable with exponents, subtract the denominator (bottom of the fraction) exponent from the numerator exponent.

   $$\frac{a^x}{a^y} = a^{x-y}$$

   (This is only true if $a \neq 0$)

   $$\frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$$

   As another example, the denominator exponent can be larger than the numerator exponent, resulting in a calculation with a negative exponent:

   $$\frac{2^2}{2^4} = 2^{2-4} = 2^{-2} = 0.25$$

   A negative exponent indicates you are to find a ***reciprocal*** of the quantity. We could also write the example above as:

   $$\frac{1}{2^{4-2}} = \frac{1}{2^2} = \frac{1}{4} = 0.25$$

   From the examples shown here, you should notice a related rule of exponents. Any factor with an exponent can be moved between the numerator and denominator of a fraction simply by changing the sign of the exponent. You will probably want to use a calculator to raise numbers to various powers.

This will be much easier than doing the repeated multiplications by hand.

4. To raise a number with an exponent to some power, multiply the exponents.

$(a^x)^y = a^{x\,y}$

$(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$

5. The product of two variables raised to a power is the same as raising each variable to the power and then finding the product.

$(a\,b)^m = a^m\,b^m$

$(2 \times 4)^3 = 2^3 \times 4^3 = 8 \times 64 = 512$

6. The ratio of two variables raised to a power is the same as the ratio of each variable raised to the power.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

(This is only true if $b \neq 0$)

$$\left(\frac{4}{2}\right)^3 = \frac{4^3}{2^3} = \frac{64}{8} = 8$$

A quantity or expression can also have an exponent, indicating the entire quantity is to be multiplied by itself: $(3x + 12)^2$ means the quantity inside the parentheses is to be multiplied by itself. Calculating *squares* and *cubes*, *square roots* and *cube roots*, are common mathematical operations.

EXPONENTIAL AND SCIENTIFIC NOTATION

Electronics measurements and calculations often involve numbers that are very large or very small. We can represent the metric prefixes as multiples of 10. It is also convenient to use multiples of 10 to represent very large and very small numbers. Any number expressed as some multiple of 10 is written in *exponential notation* (sometimes called *engineering notation*) because the 10 is written with an *exponent*. This exponent, often called a power of 10, represents how many times the number is multiplied by 10 to write it in *expanded notation*, or the "normal" format.

We can write 250000 in exponential notation as 25×10^4. All we had to do here was replace the four zeros with "$\times 10^4$." As another example, we can write 0.000025 as 25×10^{-6}. In this case we would have to divide 25 by 10 six times. A negative exponent means divide. If you move the decimal point to the right when you write the number in exponential notation, then use a negative exponent.

A number expressed with a single digit to the left of the decimal point and a power of 10 is written in *scientific notation*. This is just a particular form of engineering notation.

We could write the speed of light as 300000000 meters per second, for example, but it is more convenient to write this number as 3×10^8 meters per second. Notice that this number indicates one significant figure. If you wanted to indicate three significant figures, for example, you would write it as 3.00×10^8 meters per second.

You may see several other forms of exponential notation. Sometimes an E is written in place of the 10. ($3.56E6 = 3.56 \times 10^6$) Other times a P is used to represent a positive power of 10 while an N represents a negative power of 10. ($3.56P6 = 3.56 \times 10^6$ and $2.44N3 = 2.44 \times 10^{-3}$)

To write any number in scientific notation, first move the decimal point so there is one nonzero digit to the left of the decimal point. Then count how many places left or right you moved the decimal point. You will use this number as the exponent in the "× 10" factor. If you moved the decimal to the left, use a positive exponent and if you moved the decimal to the right, use a negative exponent.

Arithmetic Operations with Scientific Notation

One advantage of writing very large and very small numbers in exponential notation is that you don't have to keep track of so many zeros during arithmetic operations. When you work with numbers written in exponential or scientific notation you must remember a few rules, however.

To **add** or **subtract**, be certain to express all the numbers with the same power of 10. Write the numbers in a column so the decimal points align. Then add or subtract the "plain number" part as you normally would. The power of 10 for your answer is the same power in which all the numbers are expressed. **Fig 4.6** shows a sample addition and a sample subtraction using exponential notation.

To **multiply** numbers using exponential notation, first multiply the "plain number" part. Next, add the exponents for the powers of 10. Your answer is the plain-number answer times a power of 10 equal to the sum of the exponents. **Fig 4.7** shows a sample multiplication using exponential notation.

The rule for division using exponential notation is similar to the multiplication rule. To **divide** numbers written in exponential notation, first divide the "plain number" parts. Then subtract the denominator power from the numerator power. (The denominator is the bottom part of a fraction and the numerator is the top part.) **Fig 4.8** shows a sample division. Notice that we moved the denominator power of 10 into the numerator and changed the sign of the exponent.

$$\begin{array}{r} 25.40 \times 10^{3} \\ 6.15 \times 10^{3} \\ +\ 0.05 \times 10^{3} \\ \hline 31.60 \times 10^{3} \end{array} \qquad \begin{array}{r} 25.40 \times 10^{-3} \\ -\ 6.15 \times 10^{-3} \\ \hline 19.25 \times 10^{-3} \end{array}$$

(A) (B)

Fig 4.6 — Examples of addition and subtraction with numbers written in exponential notation. Be sure all the numbers have the same power of 10, and then write the numbers so the decimal points align. Add or subtract the number part, and use the common power of 10 with the answer.

$$\begin{array}{r} 25.40 \times 10^{3} \\ \times\ 6.15 \times 10^{3} \\ \hline 12700 \\ 2540 \\ 15240 \\ \hline 156.2100 \times 10^{6} \\ 156 \times 10^{6} \end{array}$$

Fig 4.7 — This example shows how to multiply two numbers using exponential notation. First multiply the number parts, then add the exponents for the powers of 10.

$$\frac{25.40 \times 10^{3}}{6.15 \times 10^{3}} = \frac{25.40 \times 10^{3} \times 10^{-3}}{6.15}$$

Fig 4.8 — This example shows how to divide two numbers using exponential notation. First divide the number parts, then subtract the denominator (the bottom part of the fraction) exponent from the numerator (the top part of the fraction) exponent. Notice in this example we moved the denominator power of 10 into the numerator and changed the sign of the exponent.

Equations

Much of algebra involves manipulating equations. We know the quantities on each side of the equal sign are equivalent. Usually the goal is to find the value of some unknown quantity. We do this by isolating that unknown quantity on one side of the equal sign, and then evaluating the expression on the other side.

Here is the most important rule to remember when you try to *solve* an equation for the unknown quantity: **Be neat!** Write each step clearly. In a jumbled mess of numbers and symbols, you will soon be hopelessly lost.

The second-most-important rule is just as significant: Anything you do to one side of the equation you must also do to the other side. A few examples will illustrate some of the most common procedures.

$2x + 4 = 8$

We can simplify this equation by subtracting 4 from both sides of the equation:

$(2x + 4) - 4 = 8 - 4$
$2x = 4$

Now we can complete the solution by dividing both sides of the equation by 2. (This is the same as multiplying both sides by the reciprocal of the term associated with the unknown, *x*.)

$$\frac{1}{2} \bullet 2x = \frac{1}{2} \bullet 4$$
$$x = 2$$

Other techniques that can be used to manipulate equations were described earlier in this chapter. See ***cross multiplication***, ***reciprocal*** and the discussion of the Laws of exponents for some important equation-solving principles.

In electronics, we often find problems in which there is more than one unknown quantity. In such cases, try to write a series of equations with the unknowns. If there are two unknown quantities, then find two equations involving those quantities. If there are three unknown quantities, find three equations, and so on. Such *systems* of *simultaneous* equations can help solve some challenging problems.

Fig 4.9 shows the schematic diagram of a simple electronics circuit. We would like to know the power dissipated in the resistor. (Power is equal to current times voltage.) In this example, we only know voltage, however. So we have two unknown quantities: current and power. Since we also know resistance, we can write a second equation from Ohm's Law, to calculate current by dividing the voltage by the resistance:

$$P = IE$$

$$I = \frac{E}{R}$$

From these two equations, we can substitute the expression for current from the second equation for current in the first equation:

$$P = \frac{E}{R}E$$

You probably recognize that E times E can be written as E^2, so we simplify this equation as:

$$P = \frac{E^2}{R}$$

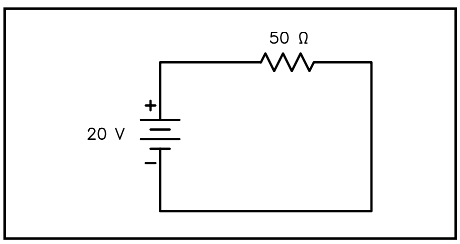

Fig 4.9 — This circuit includes a 20-V battery and a 50-Ω resistor. The text explains how we can calculate the power dissipated in the resistor.

Now it is a simple matter to fill in the known quantities of voltage and resistance to calculate power:

$$P = \frac{(20\,V)^2}{50\,\Omega} = \frac{400\,V^2}{50\,\Omega} = 8\,W$$

This example illustrates several important techniques. First, we used *substitution* to solve this problem. We substituted one expression for an equal quantity. We also used *literal equations* to solve the problem. This means we used letter symbols to represent the quantities until the last step. We could have put numbers in the equations right at the beginning, but it is often easier (and there are fewer opportunities to copy a number incorrectly) to use letter symbols. Finally, we used *dimensional analysis* with the calculation. That means we included the units associated with each measurement, and performed all the algebra operations on the units as well as the numbers. You can see this because we have volts squared in the numerator. You should check **Table 4.5** to see that volts squared divided by ohms is equivalent to watts.

Dimensional analysis is a very helpful mathematical tool if you take advantage of it. You can often use this method to help you remember the proper equation for a calculation. For example when you know that the unit of a watt can be expressed as an amp times a volt (see Table 4.5) you can write an equation that gives power as current times voltage. You can also use dimensional analysis to write a power equation involving current and resistance or voltage and resistance. Try writing these equations with the help of Table 4.5.

Linear equations involve only unknown terms with exponents no larger than 1. For any value of one variable (x) there is a corresponding value for the second variable (y). A graph of such an equation will often help you visualize the relationship between variables. For example, if x represents the current through a circuit, y might represent the voltage across a resistor. A general expression of a linear equation is:

$$y = mx + b$$

where m represents the *slope* of the line (the change in y divided by the corresponding change in the x variable) often written as

$$\frac{\Delta y}{\Delta x}$$

and b represents the *y intercept*, or the point where the line crosses the vertical axis when $x = 0$.

An equation that involves a variable term with an exponent of 2 (a squared term) is called a *quadratic equation*. The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

where a, b and c are constant terms, or values for a particular equation, and x is the variable quantity.

Quadratic equations always have two solutions, which means there are two values for x that satisfy the equation. Perhaps the most straightforward way to solve a quadratic equation for the unknown quantity is to use the *quadratic formula*. This formula can be used to solve any quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the plus or minus symbol in front of the radical sign. This tells us that one solution requires that we add the resulting term to –b and the other solution requires that we subtract the resulting term from –b. This comes about because when you square a negative number, you get a positive result. There are always two solutions to a square root. A simple example will illustrate the use of the quadratic formula.

$2x^2 + 4x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - (4 \times 2 \times (-6))}}{2 \times 2}$$

$$x = \frac{-4 \pm \sqrt{16 - (-48)}}{4}$$

$$x = \frac{-4 \pm \sqrt{64}}{4} = \frac{-4 \pm 8}{4}$$

$$x = \frac{-4 + 8}{4} = \frac{4}{4} = 1$$

$$x = \frac{-4 - 8}{4} = \frac{-12}{4} = -3$$

Measurement Units and Constants

Nearly every time we use a number, it represents some measured physical quantity. We might use a measuring tape to find the correct length of wire for an antenna, or an ohmmeter to measure the value of a resistor.

Each measurement includes a number representing its size and a unit that allows it to be compared with other measurements of a similar type. One dipole may be 126 ft, 6 inches long and another dipole may be 65 ft, 11 inches long, for example. The units used with any measurement represent standards that are generally accepted, so meaningful comparisons can be made. These comparisons are only meaningful if the measurements use the same units. It is difficult to compare a dipole that is 65 ft 11 inches long with one specified as a 40-m dipole, because the units are not the same.

US CUSTOMARY SYSTEM

Most US residents are familiar with units like the inch, foot, quart, gallon, ounce and pound. These units represent standard measurement values used in the *US Customary* measuring system. **Table 4.3** lists some common US Customary units, and some not-so-common ones. This table shows the relationships between various *linear*, *area*, *liquid volume*, *dry volume* and *weight* measurements.

The primary disadvantage of the US Customary measuring system is that there is no logical relationship between various-sized units of a similar type. Most electronics measurements are made in the internationally accepted *metric* system, for this reason.

METRIC SYSTEM

In the metric system, measuring units are always a multiple of 10 times larger or smaller than other units of the same type. Metric-system measurements are always based on a measurement unit and a set of prefixes to describe the larger and smaller variations of that unit. For example, a millimeter is ten times smaller than a centimeter, a meter is a hundred times larger than a centimeter and a kilometer is a thousand times larger than a meter. (In nearly every country of the world except the US, this unit of distance measurement is spelled *metre*, which helps distinguish the distance unit from an electrical measuring instrument, also called a meter.) **Table 4.4** shows the common

Table 4.3
US Customary Units

Linear Units
12 inches (in) = 1 foot (ft)
36 inches = 3 feet = 1 yard (yd)
1 rod = $5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet
1 statute mile = 1760 yards = 5280 feet
1 nautical mile = 6076.11549 feet

Area
1 ft^2 = 144 in^2
1 yd^2 = 9 ft^2 = 1296 in^2
1 rod^2 = $30\frac{1}{4}$ yd^2
1 acre = 4840 yd^2 = 43,560 ft^2
1 acre = 160 rod^2
1 $mile^2$ = 640 acres

Volume
1 ft^3 = 1728 in^3
1 yd^3 = 27 ft^3

Liquid Volume Measure
1 fluid ounce (fl oz) = 8 fluidrams = 1.804 in^3
1 pint (pt) = 16 fl oz
1 quart (qt) = 2 pt = 32 fl oz = $57\frac{3}{4}$ in^3
1 gallon (gal) = 4 qt = 231 in^3
1 barrel = $31\frac{1}{2}$ gal

Dry Volume Measure
1 quart (qt) = 2 pints (pt) = 67.2 in^3
1 peck = 8 qt
1 bushel = 4 pecks = 2150.42 in^3

Avoirdupois Weight
1 dram (dr) = 27.343 grains (gr) or (gr a)
1 ounce (oz) = 437.5 gr
1 pound (lb) = 16 oz = 7000 gr
1 short ton = 2000 lb, 1 long ton = 2240 lb

Troy Weight
1 grain troy (gr t) = 1 grain avoirdupois
1 pennyweight (dwt) or (pwt) = 24 gr t
1 ounce troy (oz t) = 480 grains
1 lb t = 12 oz t = 5760 grains

Apothecaries' Weight
1 grain apothecaries' (gr ap) = 1 gr t = 1 gr a
1 dram ap (dr ap) = 60 gr
1 oz ap = 1 oz t = 8 dr ap = 480 gr
1 lb ap = 1 lb t = 12 oz ap = 5760 gr

metric prefixes, their abbreviations and the multiplication factor associated with each one.

SI

The metric units make up an internationally recognized measuring system used by most scientists throughout the world. We call this the International System of Units, abbreviated SI (for the French, Système International d'Unités).

By the late 1700s, scientists were developing this measuring system based on multiples of ten. The original intent was to develop a measuring system based on measuring units that could be reproduced as needed. The meter was first defined as one ten millionth of the distance between the equator and the north pole, as measured along the longitude line running through Paris, France. A kilogram was originally defined as the mass of 1 liter (spelled *litre* throughout the rest of the world) of water at 4°C. As measuring instruments improve, the definitions are revised to reflect the greater measuring accuracy. For example, in 1960 the definition of a meter was revised to be a multiple of the wavelength of a particular orange-red light wave.

Table 4.4
Metric Prefixes

Prefix	*Symbol*		*Multiplication Factor*
exa	E	10^{18} =	1,000,000,000,000,000,000
peta	P	10^{15} =	1,000,000,000,000,000
tera	T	10^{12} =	1,000,000,000,000
giga	G	10^{9} =	1,000,000,000
mega	M	10^{6} =	1,000,000
kilo	k	10^{3} =	1,000
hecto	h	10^{2} =	100
deca	da	10^{1} =	10
(unit)		10^{0} =	1
deci	d	10^{-1} =	0.1
centi	c	10^{-2} =	0.01
milli	m	10^{-3} =	0.001
micro	µ	10^{-6} =	0.000001
nano	n	10^{-9} =	0.000000001
pico	p	10^{-12} =	0.000000000001
femto	f	10^{-15} =	0.000000000000001
atto	a	10^{-18} =	0.000000000000000001

The metric system is based on the definitions of certain fundamental units, with all other units being based on those units. These fundamental, or defined units represent length (meter), mass — you might think of this as somewhat equivalent to weight — (gram), time (second), thermodynamic temperature (kelvin or degree celsius), luminous intensity of light (candela), the amount of substance — a measure of the number of atoms or molecules — (mole) and electric current (ampere).

All other units represent combinations of these fundamental units. For example, the unit of power (watt) is a measure of the energy required to move a one kilogram object a vertical distance of one meter in one second. **Table 4.5** lists some common units and their expression in terms of the base SI units.

Suppose you measure the frequency of a radio wave as 3825000 hertz. If we move the decimal point six places to the left, we would write this frequency as 3.825×10^6 hertz. Looking at Table 4.4, you can replace the "$\times 10^6$" part with the prefix mega. So we can write this frequency as 3.825 MHz (using the abbreviations M for mega and Hz for hertz). Similarly, you can use other metric-system prefixes to replace powers of 10 in large and small numbers.

As another example, suppose you find a capacitor marked with a value of 25 microfarads or 25 µF. From Table 4.4, you can find the multiplication factor of 10^{-6} for the prefix micro. That means you can write this capacitor value as 25×10^{-6} farads, or 0.000025 F.

CONVERTING BETWEEN US CUSTOMARY AND METRIC SYSTEMS

Sometimes it is convenient to convert between these two common measuring systems. You may know an antenna length in meters, but want to use your tape measure marked in feet and inches to cut the antenna, for example. **Table 4.6** lists most of the conversion factors you will ever need.

Using Proportions to Solve Conversion Problems

To solve US Customary and metric conversions we will use proportions. This method also illustrates some other very useful mathematical tools. The advantage of using proportions to solve conversions is that you never have to figure out if you must multiply or divide. The proportion shows you what to do!

Table 4.5

SI Fundamental Units

Quantity	*Unit Name*	*Symbol*	*In Terms of Other Units*	*In terms of SI Base Units*
Distance, length	meter	m		m
Mass	kilogram	kg		kg
Time	second	s		s
Thermodynamic temperature	kelvin	K		K
Luminous intensity	candela	cd		cd
Amount of substance	mole	mol		mol
Electric current	ampere	A		A

SI Derived Units

Quantity	*Unit Name*	*Symbol*	*In Terms of Other Units*	*In terms of SI Base Units*
Force, pressure	newton	N		$\frac{m\,kg}{s^2}$
Energy, work	joule	J	N m, $\Omega A^2 s$	$\frac{m^2\,kg}{s^2}$
Frequency	hertz	Hz		$\frac{cycles}{s}$
Power	watt	W	$\frac{V^2}{\Omega}$, A V	$\frac{m^2\,kg}{s^3}$
Electric charge, quantity of electricity	coulomb	C		s A
Electromotive force, voltage	volt	V	$A\,\Omega, \frac{W}{A}$	$\frac{m^2\,kg}{s^3\,A}$
Electric resistance	ohm	Ω	$\frac{V}{A}$	$\frac{m^2\,kg}{s^3\,A^2}$
Electric conductance	siemens	S	$\frac{A}{V}$	$\frac{s^3\,A^2}{m^2\,kg}$
Capacitance	farad	F	$\frac{C}{V}$	$\frac{s^4 A^2}{m^2\,kg}$
Inductance	henry	H	$\frac{V\,s}{A}$	$\frac{m^2\,kg}{s^2\,A^2}$

Table 4.6

Metric Conversion Factor		**US Customary Conversion Factor**
(Length)		
25.4	mm	1 inch
2.54	cm	1 inch
30.48	cm	1 foot
0.3048	m	1 foot
0.9144	m	1 yard
1.609	km	1 mile
1.852	km	1 nautical mile
(Area)		
645.16	mm^2	1 $inch^2$
6.4516	cm^2	1 in^2
929.03	cm^2	1 ft^2
0.0929	m^2	1 ft^2
8361.3	cm^2	1 yd^2
0.83613	m^2	1 yd^2
4047	m^2	1 acre
2.59	km^2	1 mi^2
(Mass)	(Avoirdupois Weight)	
0.0648	grams	1 grains
28.349	g	1 oz
453.59	g	1 lb
0.45359	kg	1 lb
0.907	tonne	1 short ton
1.016	tonne	1 long ton
(Volume)		
16387.064	mm^3	1 in^3
16.387	cm^3	1 in^3
0.028316	m^3	1 ft^3
0.764555	m^3	1 yd^3
16.387	ml	1 in^3
29.57	ml	1 fl oz
473	ml	1 pint
946.333	ml	1 quart
28.32	l	1 ft^3
0.9463	l	1 quart
3.785	l	1 gallon
1.101	l	1 dry quart
8.809	l	1 peck
35.238	l	1 bushel
(Mass)	(Troy Weight)	
31.103	g	1 oz t
373.248	g	1 lb t
(Mass)	(Apothecaries' Weight)	
3.387	g	1 dr ap
31.103	g	1 oz ap
373.248	g	1 lb ap

To set up a conversion between metric and US Customary units, just make a ratio of the conversion factors and another of the measurements. Set the two ratios equal to each other and solve the proportion for the unknown measurement.

$$\frac{\text{metric conversion}}{\text{US conversion}} = \frac{\text{metric measurement}}{\text{US measurement}}$$

Suppose we know that a dipole antenna is 126 ft 6 inches long. (For simplicity, change that to 126.5 ft.) How long is this antenna in meters? Look down the *Metric Unit* column of Table 4.6 until you find an “m” for “meters.” Then go across to the *US Unit* column to find “foot.” Notice there are two conversion factors for meters; one for “foot” and another for “yard.” When you’ve located the proper conversion factor, you will see there is 0.3048 meter in 1 ft. (A meter is a little more than 3 ft.) We know the US measurement in this case, and want to find the metric measurement.

$$\frac{0.3048\,\text{m}}{1\,\text{ft}} = \frac{\text{metric measurement}}{126.5\,\text{ft}}$$

In this example we will ***cross multiply*** the US measurement term to leave the unknown measurement by itself on one side of the equal sign. When you cross multiply, just take any part of the proportion diagonally across the equal sign.

$$\frac{0.3048\,\text{m} \times 126.5\,\text{ft}}{1\,\text{ft}} = \text{metric measurement}$$

$$= \frac{38.56\,\text{m ft}}{1\,\text{ft}} = 38.56\,\text{m}$$

Notice that we include the appropriate units with the metric and US conversion factors. After we cross multiply, the units of feet that go with the dipole length cancel with the units of feet that go with the US conversion factor, leaving only units of meters in our answer. *Dimensional analysis* ensures we have solved the proportion properly.

Trigonometry

Trigonometry refers to the mathematics of angles, especially as they relate to triangles. When two lines meet or cross, they form angles. The point where the lines meet or cross is called the *vertex* of the angle. We usually measure an angle as an arc of a circle across the smallest opening between the lines. We can describe an angle as falling in one of three categories. **Fig 4.10** shows a *right angle*, an *acute angle* and an *obtuse angle*.

The figure shows the angles in terms of a degree measurement. A degree is $1/360^{th}$ of a circle, or $1/360^{th}$ of a complete revolution. We can also measure angles in radians. A radian is an angle measure obtained by taking the length of the radius of a circle and laying that length along the circumference of that circle. See **Fig 4.11**. There are 2π radians in one circle, or 360°. Two useful conversion relationships are:

$1° = 1.745 \times 10^{-2}$ radians, and:
1 radian = 57.296°

When three lines cross in such a manner that they form three angles, these lines form a *triangle*. We normally identify a triangle by the largest angle that it includes. This means there are three types of triangles. **Fig 4.12** shows examples of the three types of triangles. In electronics, we will use right triangles in a variety of calculations.

If you add the three angles in a triangle, you will always get a total of 180°. For a right triangle, then, the sum of the other two angles must be 90°.

The triangles shown in Fig 4.12 have their angles labeled with upper-case letters and their sides labeled with corresponding lower-case letters. Notice that the side opposite each angle uses the lower-case letter of its opposite angle. With a right triangle, the side opposite the right angle has a special name. It is called the *hypotenuse* of the right triangle. In Figure 4-12B, side a is *opposite* angle A, side b is *opposite* angle B and the hypotenuse (side c) is opposite the right

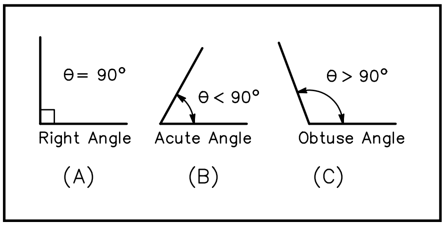

Fig 4.10 — Three types of angles. A shows a right angle, B shows an acute angle and C shows an obtuse angle.

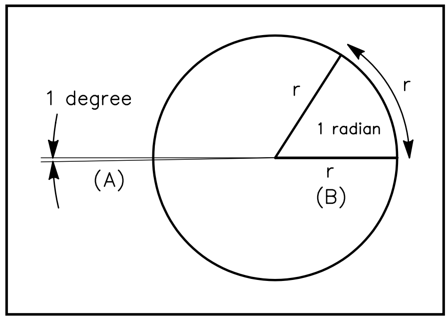

Fig 4.11 — Part A illustrates the measure of 1° as part of a circle. Part B shows the measure of 1 radian as part of a circle.

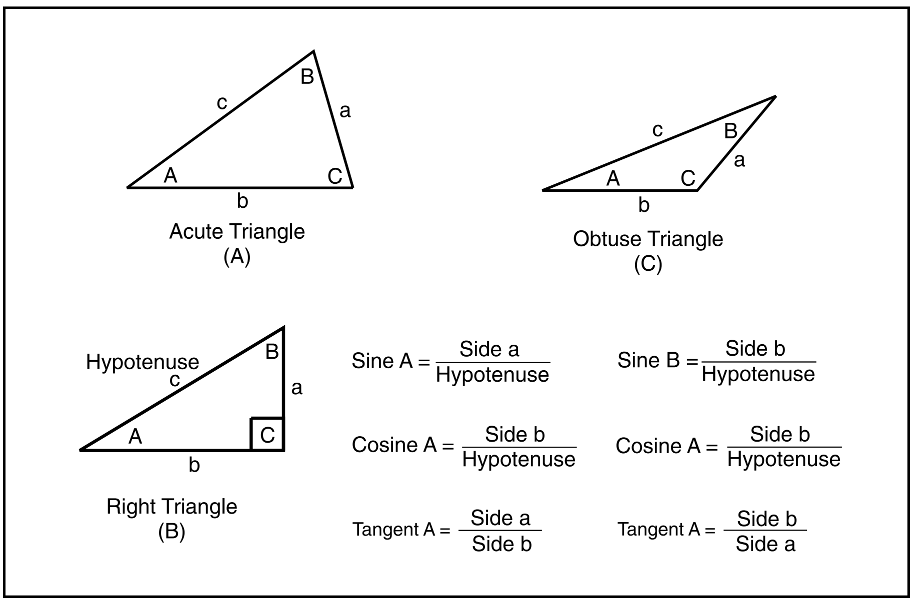

Fig 4.12 — Part A shows an acute triangle, Part B shows a right triangle and Part C shows an obtuse triangle.

angle (angle C). We can also say that side a is *adjacent* to angle B and side b is *adjacent* to angle A. (The hypotenuse is also adjacent to both angles A and B, but since the hypotenuse is otherwise uniquely identified we don't use this name for the hypotenuse.) The Greek letters theta (θ) and phi (ϕ) are often used to represent angles. Sometimes you will also see the Greek letters alpha (α), beta (β) and gamma (γ) used to represent the angles in a triangle.

WORKING WITH RIGHT TRIANGLES

Trigonometry defines relationships between the lengths of the sides of a right triangle and its angles. With these relationships and any combination of three sides or angles we can calculate any of the quantities we don't know. For example, if you know two sides and one angle, you can calculate the third side and the other two angles. While there are six functions defined for any right triangle, you can perform any required calculations if you know three of those functions.

The three functions we will use are the sine, cosine and tangent. Each function is defined in terms of an angle and two sides of the triangle.

$$\text{sine}\,\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\text{cosine}\,\theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\text{tangent}\,\theta = \frac{\text{side opposite}}{\text{side adjacent}}$$

Fig 4.12B shows the definitions of the three important trigonometry functions associated with angles A and B.

These functions are usually abbreviated as *sin*, *cos* and *tan*. Each function represents a ratio of two sides of the triangle, and this ratio is the same for any given angle, no matter how large or small the triangle. For example, **Fig 4.13** shows two right triangles that each include a 30° angle. The sine of the 30° angle is 0.5 no matter which triangle we are working with. Likewise, each of these triangles also includes a 60° angle, and the sine of the 60° angle is always 0.866.

Most scientific calculators include keys to find these function values. It is important to know if the calculator will understand the angle you enter as measured in degrees or radians. Most calculators work with angles measured in degrees, but some use radians. Some calculators will also use radians if you enter the proper keystrokes before starting. Computers usually work with angles measured in radians.

Suppose you know the ratio of sides, and want to know what angle is associated with that value. This is a question of finding the *inverse function*. Suppose you know that the side opposite an angle divided by the hypotenuse equals 0.5. What is the angle? Since opposite over hypotenuse is the definition of sine, we want to find the inverse sine, or arcsine (often abbreviated *arcsin*). In this example, the answer is 30°. Likewise, we can also find the arccosine (*arccos*) and arctangent (*arctan*) of an angle. You will also see these inverse functions written as $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Here the –1 exponent is simply a short-hand notation to indicate the inverse function. Do not try to follow the rules of significant figures when you find the value of a trigonometry function or its inverse function. Do follow the rules when you calcu-

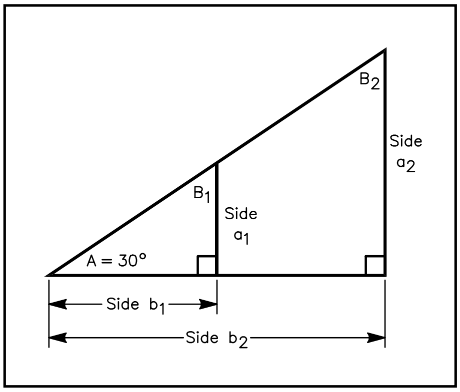

Fig 4.13 — This drawing shows two right triangles that each include a 30° angle and a 60° angle. Notice that values of the trigonometry functions don't depend on how long the sides of the triangles are; the same values of sin, cos and tan apply to each of these triangles.

late the sides and angles of a triangle, however. For example, if you know an angle measurement to three or four significant figures, express the other angles you calculate with the same number of significant figures. If you know the length of a side to four significant figures, express the calculated sides to four significant figures.

The sine and cosine functions are used in many ways in electronics. You will often see graphs of these functions used to represent the waveform of an alternating current signal. **Fig 4.14** shows graphs of the sine, cosine and tangent functions for angles from 0 to 360°. (The horizontal axis on the graph is also marked in radians, from 0 to 2π radians.)

In addition to the three trigonometry functions described here, there is one other very important relationship for working with right triangles. This principle was discovered by a Greek mathematician, Pythagoras. The *Pythagorean Theorem* states that the square of the hypotenuse is equal to the sum of the squares of the other two sides. Written as an equation, this is:

$$c^2 = a^2 + b^2$$

We can take the square root of both sides to solve this equation for the hypotenuse.

$$c = \sqrt{a^2 + b^2}$$

We can also solve the original equation for either of the other sides and then take the square root.

$$a^2 = c^2 - b^2$$

$$a = \sqrt{c^2 - b^2}$$

There is no single correct procedure for calculating the parts of a

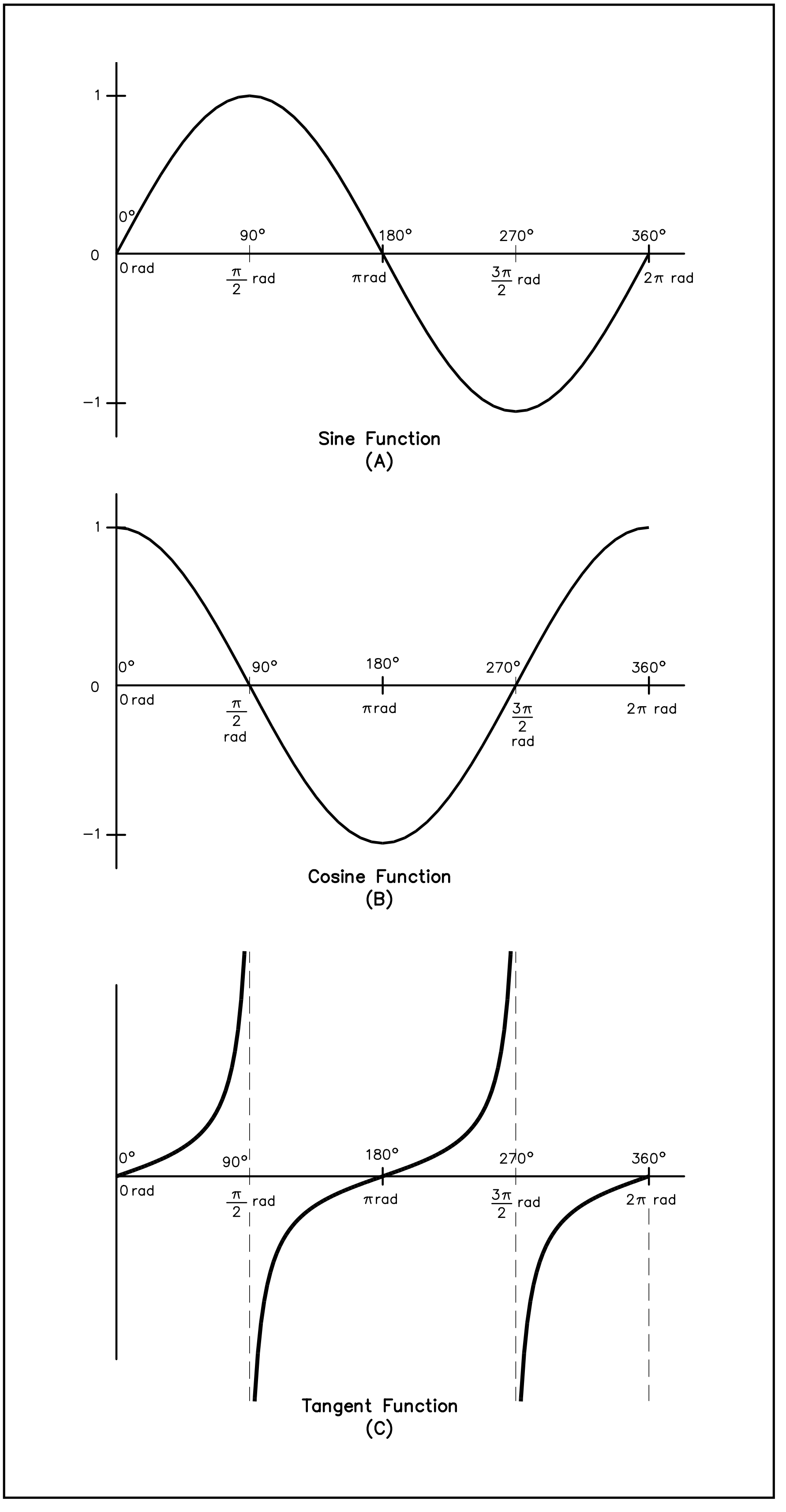

Fig 4.14 — Part A shows a graph of the sine function for angles from 0 to 360°. Part B is a graph of the cosine function and Part C is a graph of the tangent function for angles from 0 to 360°. (Note that the horizontal axis also indicates angles in radians.)

right triangle. Select a "trig" function or the Pythagorean Theorem depending on what parts of the triangle you know. **Fig 4.15** shows a right triangle. The drawing shows the two sides, and you have to calculate the hypotenuse and the two angles. We could use the Pythagorean Theorem to calculate the hypotenuse, but let's use the tangent function to find angle A first.

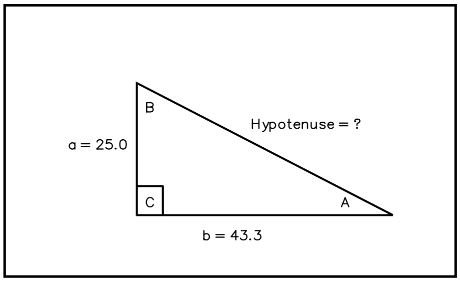

Fig 4.15 — Find the hypotenuse and the two acute angles of this right triangle. You can use any of the "trig" functions and the Pythagorean Theorem.

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}}$$

$$\tan A = \frac{25.0}{43.3} = 0.577$$

Next we must find the arctangent of this ratio:

A = 30.0°

Since the two acute angles of a right triangle must add up to 90°, we can see that angle B must be 60.0°. Then we can use the sine, cosine or Pythagorean Theorem to calculate the hypotenuse. Let's use the sine function for this example. (You can use the cosine to verify that it gives the same answer.)

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}}$$

Cross multiply to solve this literal equation for the hypotenuse, and calculate the value.

$$\text{hypotenuse} = \frac{\text{side opposite}}{\sin A}$$

$$\text{hypotenuse} = \frac{25.0}{\sin 30°} = \frac{25.0}{0.500} = 50.0$$

As an example of using the Pythagorean Theorem, we will use that to solve for hypotenuse of this triangle also.

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{25.0^2 + 43.3^2} = \sqrt{625 + 1870}$$

$$c = \sqrt{25\bar{0}0} = 50.0$$

Notice that we have followed the rules for significant figures through these examples. In this last step, we had to round off the 43.3^2 term. Then the addition term under the radical was limited to the hundreds place, or three significant figures.

WORKING WITH ACUTE AND OBTUSE TRIANGLES

All the trigonometry functions and techniques described in the last section apply *only* to *right triangles*. You may occasionally need to work with an acute or obtuse triangle. In this case it will be handy to remember the *Law of Sines* and the *Law of Cosines*. The Law of Sines tells us that the length of any side is proportional to the sine of the opposite angle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If you know one of the angles, the side opposite that angle, and one other side or angle, you can use this relationship to calculate the fourth side or angle. The Law of Sines is a simple proportion, and can be solved for the unknown quantity using cross multiplication. See **Fig 4.16**.

The Law of Cosines equation will remind you a bit of the Pythagorean Theorem. You can find any side of the triangle if you know the other two sides and the angle opposite the unknown side. (This is just the opposite of the Law of Sines, where you must know one angle and its opposite side.)

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

We could write similar equations solved for b^2 and c^2, but that isn't necessary, since it really doesn't matter which side is labeled a, b or c as long as each angle and its side opposite use the same letter. We can take the square root of both sides of this equation to solve for a:

$$a = \sqrt{b^2 + c^2 - 2bc\cos(A)}$$

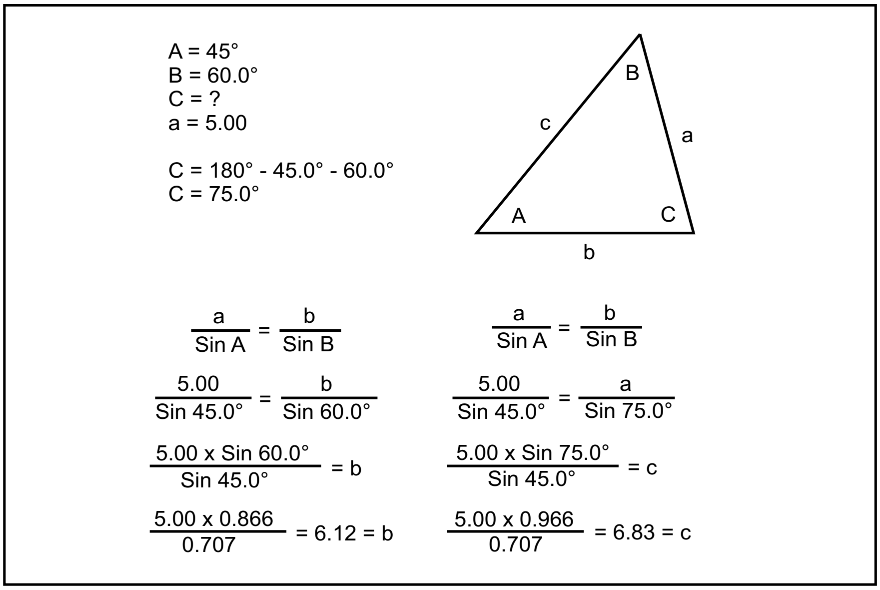

Fig 4.16 — The Law of Sines is used to calculate sides b and c of this acute triangle.

Coordinate Systems

A coordinate system helps us draw graphs to represent quantities and equations. A coordinate system provides a scale with a set of numbers to represent the location of a point on a surface. There are several such coordinate systems used in electronics. In this section we will briefly discuss three coordinate systems: the *rectangular*, or *cartesian coordinate system*, the *polar coordinate system* and the *spherical coordinate system*.

RECTANGULAR COORDINATES

Fig 4.17 shows a portion of a *rectangular*, or *cartesian coordinate system*. It is simply a pair of number lines that cross at a 90° angle. The scale on the number lines is chosen to suit the particular needs of any given situation. The graduations on one scale can be larger than the other, one or both lines can be far from 0, with an arbitrary crossing point to show the region of interest. This coordinate system represents a plane, two-dimensional surface. You have probably used graph paper drawn as a rectangular coordinate system.

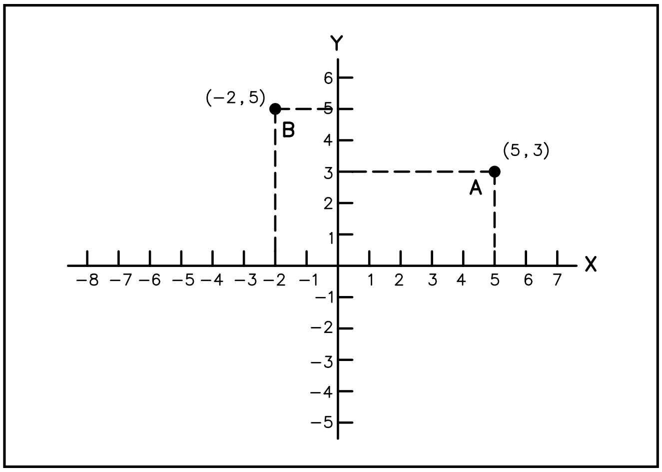

Fig 4.17 — A set of horizontal and vertical lines, marked off with a number scale, forms a rectangular coordinate system. We usually label the horizontal line, or axis, the X axis, and the vertical line the Y axis. Any point on the surface can be identified with a pair of numbers, representing the distance along the X axis and the distance along the Y axis to reach the point. When the point is identified with a pair of numbers, the convention is to list the X value first, then the Y value, as (X, Y).

The horizontal line, or *axis*, is often labeled X. This usually represents the independent, or controlled variable when an equation is being graphed.

The vertical line is often labeled Y. This usually represents the dependent variable (the value depends on the conditions set for the controlled variable).

Any point on a rectangular coordinate system can be specified by a pair of numbers, such as (–2, 5) or (5, 3). These numbers represent the distance along the X axis and the distance along the Y axis to reach the point.

POLAR COORDINATES

We specify the distance to a point with measurements along the X and Y axes with a rectangular coordinate system. Sometimes it is more convenient to specify the shortest distance from the center or *origin* to the point. In that case we can use a *polar coordinate system*. **Fig 4.18** shows an example of this system. The lines help mark the center of the system and provide a reference by dividing the circle into four equal parts, but they are not really necessary. Again we specify the location of any point on the surface with a pair of numbers, but this time the numbers represent the direct distance from the

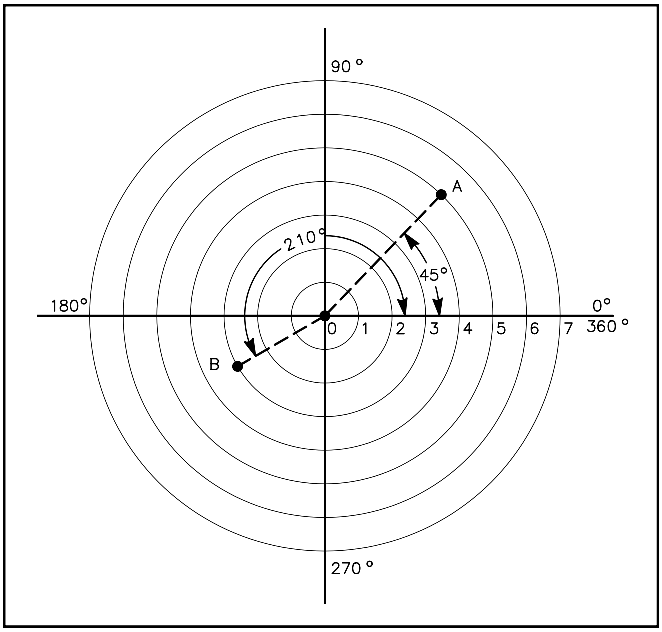

Fig 4.18 — This drawing shows a polar coordinate system. Any point on this surface is identified with a pair of numbers representing the distance from the origin directly to the point, and an angle or direction.

origin to the point, and an angle. The angle is usually measured counterclockwise from the line extending to the right side. The distance represents the *magnitude* or length of the value, measured as a straight line (the shortest distance) from the center to the point.

The circles represent increasing distances from the origin. You can choose any convenient scale for the radius of these circles. You don't always need the complete circles. Often you will only need one quarter or one half of the circle. You can buy graph paper marked off with a polar coordinate system, although you seldom need such graph paper. You can even use rectangular coordinate graph paper or plain paper, with a drawing compass, ruler and protractor if you do want a scale drawing.

Many times you will have to convert between rectangular and polar coordinate systems. You will find the trigonometry functions especially helpful at such times. **Fig 4.19** shows a right triangle drawn to illustrate such a conversion. The sides of the triangle represent the X (4) and Y (3) values of a rectangular coordinate system. The hypotenuse of the right triangle represents the distance between the origin and the end point on a polar coordinate system. Angle A represents the polar-coordinate angle.

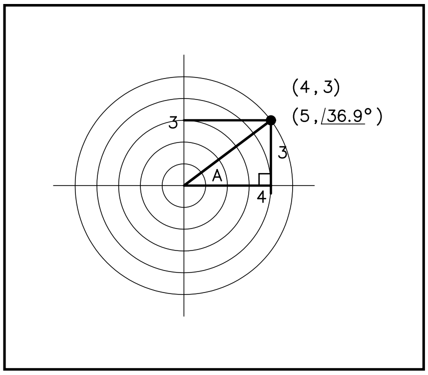

Fig 4.19 — The right triangle on this graph shows how we can specify the same point in rectangular coordinates and in polar coordinates. The rectangular coordinates (4, 3) and the polar coordinates (5, ∠36.9°) both represent the same point on this graph. You can use the trigonometry functions discussed earlier to convert between these two systems.

You should be able to use the various "trig" functions and the Pythagorean Theorem to calculate the hypotenuse (5) and angle A (36.9°) for this problem. If you knew the hypotenuse and angle A you should also be able to calculate the other two sides of the triangle, to convert from polar to rectangular coordinates.

SPHERICAL COORDINATES

Both of the coordinate systems described above represent a two-dimensional surface. This is fine for most electronics problems, but occasionally it is helpful to have a three-dimensional coordinate system. It is possible to add a third axis to the rectangular coordinates, which forms a 90° angle with the other two. (Look at the corner of a room, with the two lines along the floor and walls representing the X and Y axes. Then the corner between the two walls represents the Z axis.) A set of three numbers (X, Y, Z) will represent any point in the three-dimensional space this system represents.

It is also possible to rotate the circles of the polar coordinate system to create a sphere. The resulting *spherical coordinate system* gives us another way to represent a point in three dimensions. In this system we use the radius, or distance from the center to the point, and two angles — one representing an angle measured "horizontally" and the other representing an angle measured "vertically." (Think about our Earth, and the way we draw lines of longitude and latitude.)

Complex Algebra

We most often use the rectangular and polar coordinate systems for electronics problems involving resistance, reactance and impedance (and conductance, susceptance and admittance, their reciprocals). When we work with these quantities we draw the resistance or conductance along the X axis and the reactance or susceptance along the Y axis. The hypotenuse represents impedance or admittance.

We must use some special mathematical techniques when working with these quantities because we must always distinguish between them. It is most convenient to use the algebra of what mathematicians call *imaginary numbers*, although there is nothing imaginary about these electronics quantities. Mathematicians use these techniques when they work with quantities that involve the square root of minus 1, written as $\sqrt{-1}$. It is impossible to find a number that when multiplied by itself gives –1, so this quantity is *imaginary*, yet it does show up in some mathematical procedures. Mathematicians represent this quantity with a lower-case italic *i*. Quantities that include real and imaginary parts are called *complex numbers*.

In electronics, we use a lower-case italic *j* to represent numbers on the reactance or susceptance line, or Y axis of a graph. The algebra of complex numbers provides a way to add, subtract, multiply and divide quantities that include both resistive and reactive components. You can best think of the *j* as an *operator* that produces a 90° rotation from the resistance line. An operator is just a mathematical procedure applied to a quantity. An exponent is an operator that tells you how many times to multiply a quantity times itself and the radical sign ($\sqrt{\ }$) is an operator that tells you to take the square root.

When you see a reactance expressed as *j*250 Ω, place this quantity along the Y axis on your graph. A reactance expressed as –*j*300 Ω tells you to rotate 90° in the clockwise direction instead of the normal counterclockwise direction.

Inductive reactance is specified with a +*j* for series circuits, because the voltage across an inductor *leads* the current through it. Since voltage and current are in phase in a resistor, the voltage across the inductor leads the voltage across the resistor by 90°. (For parallel circuits, the voltage across the resistor and inductor is the same, so they are in phase. In that case the current through the inductor *lags* the current through the resistor, so the current associated with the inductive reactance gets a –*j* operator.)

Capacitive reactance is just the opposite. It is specified with a –*j* operator for series circuits because the voltage across a capacitor lags the current, so the current across the capacitor *lags* the voltage across the resistor — or is 90° behind. (For parallel circuits, the same voltage is applied to the resistor and capacitor, so the capacitor current *leads* the resistor current. The current associated with the capacitive reactance gets a +*j* operator for parallel circuits.)

A handy memory device for these relationships is the saying, "ELI the ICE man." The E represents voltage, I represents current, L is inductance and C is capacitance.

Impedance is a combination of resistance and reactance. When we specify a series-circuit impedance as 50 + *j*200 Ω, you know this represents a circuit with a 50-Ω resistance in series with a 200-Ω inductive reactance. Likewise, an impedance of 50 – *j*200 Ω represents a circuit with a 50-Ω resistance in series with a 200-Ω capacitive reactance.

Both of these impedances can be expressed in polar-coordinate

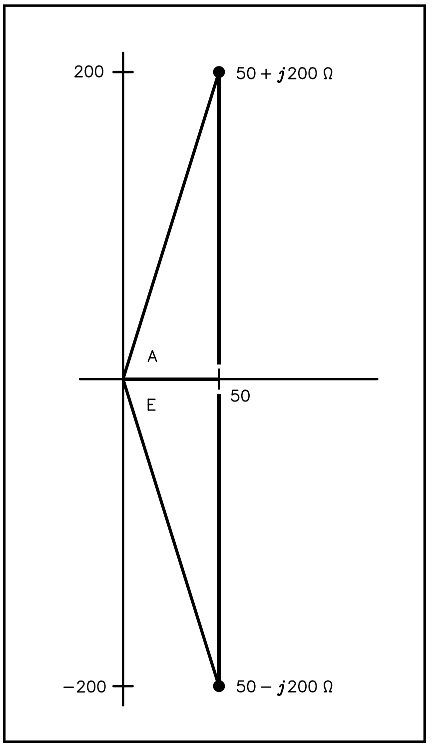

Fig 4.20 — The two triangles shown on this graph represent the impedances of two circuits. Triangle A represents a 50-Ω resistance in series with a 200-Ω inductive reactance (50 + *j*200 Ω). Triangle B represents a 50-Ω resistance in series with a 200-Ω capacitive reactance (50 – *j*200 Ω).

form. Plot the values on a graph and calculate the hypotenuse of the right triangle and angle A as shown on **Fig 4.20**. Then you can write these impedances as 206 Ω ∠76° and 206 Ω ∠–76°.

RULES FOR WORKING WITH COMPLEX NUMBERS

Addition and subtraction of complex numbers are best done using rectangular-coordinate form. When you add complex numbers written in rectangular notation you add the parts along the X axis, and you add the parts along the Y axis. The result gives a new set of X, Y coordinates, representing the addition of the two complex values. To subtract complex numbers you subtract one X part from the other, and subtract the corresponding Y parts. The result is a new set of X, Y coordinates, representing the subtraction of the two values.

For example, what is the total impedance of a circuit that has an impedance of 30 + *j*150 Ω in series with an impedance of 40 – *j*100 Ω? We can write this addition as:

$$\begin{array}{r} 30 + j\,150\ \Omega \\ +\ 40 - j\,100\ \Omega \\ \hline 70 + j\ \ 50\ \Omega \end{array}$$

If you have to add or subtract impedances given in polar notation (a magnitude or length and an angle), first convert these values to rectangular-coordinate form. Use the trigonometry functions and Pythagorean Theorem described earlier in this chapter. If you need the answer specified in polar-coordinate form you can convert back to that notation after performing the addition.

Multiplication and division of complex numbers is best done in polar-coordinate form. When you multiply complex numbers in polar-coordinate form, you multiply the magnitudes and add the angles. When you divide complex numbers in polar notation you divide the magnitudes and subtract the angles.

Suppose you want to find the impedance of a circuit that has a resistor in parallel with a capacitor. When you apply 10.0 V to the circuit, you measure 0.250 A of current. You measure the phase angle between the current and the voltage, and find the current leads the voltage by 30.0°. We can calculate the impedance of this circuit (represented by a capital Z) using Ohm’s Law.

$$Z = \frac{E}{I}$$

$$Z = \frac{10.0\ \text{V} \angle 0.0°}{0.250\ \text{A} \angle 30.0°}$$

The components are in parallel, so the same voltage is applied to both the resistor and capacitor. Use the voltage as the phase reference for parallel-circuit calculations, so the voltage has a 0° phase angle. To perform this division, first divide the voltage magnitude by the current magnitude. Then subtract the denominator phase angle from the numerator phase angle.

$$\frac{10.0\ \text{V}}{0.250\ \text{A}} = 40.0\ \Omega$$

and

$$0.0° - (30.0°) = -30.0°$$

These two values specify the impedance of the circuit in this example. We can put them together and write the circuit impedance in polar-coordinate form as:

$$40.0\,\Omega \angle -30.0°$$

The negative phase angle tells us there is a capacitive reactance as part of the impedance.

Logarithms

A logarithm is an exponent. *Common logarithms* use the number 10 as their *base*. You have some experience with "powers of 10" from writing numbers in exponential or scientific notation.

A common log, as it is usually called, is the exponent or power to which you must raise 10 to get a certain number. In the examples above, we raised 10 to the third power to get 1000. The log of 1000, then, is 3. The log of 1000000 is 6. In general, we define a common logarithm with two equations. If:

$N = 10^x$, then:
$\log(N) = x$

Sometimes you will see this written as $\log_{10}(N) = x$. This is simply to ensure that you know the base of the logarithm is 10.

Finding the log of a multiple of 10 is easy, as these examples show. You may wonder to what power you can raise 10 to get a number like 2. That is a good question, and the answer is 0.301. Logs are usually decimal fractions rather than whole numbers. Logs for numbers smaller than 10 are less than 1; logs for numbers larger than 10 are greater than 1. From the definition of a log, we can write the expression:

$2 = 10^{0.301}$

The easiest way to find any logarithm is with your calculator. Simply enter the number whose log you want to find, and then push the button labeled "log." It is easy to find that log (5) = 0.699, for example.

It is interesting to note that log (1) = 0, because anything (including 10) raised to the zero power is 1. The log of 0 is undefined, because there is no power to which you can raise 10 and get 0.

The inverse log is called the *antilog* (often written $\log^{-1}$). When we know the log and want to find the original number, we want the antilog. To find an antilog, simply raise 10 to the given power. Your calculator probably has a button labeled "10^x" or something similar. What is the antilog of 1.845? $10^{1.845} = 70$. Don't try to follow the rules for significant figures when finding logs or antilogs. Do follow the rules with the values you calculate from logs and antilogs, however.

The second base that is frequently used for logarithms is a number usually represented by *e*. (Sometimes the Greek letter epsilon (ε) is used to represent *e* although this is an incorrect representation.) This number is approximately 2.71828. This is not an exact value, because the decimal fraction doesn't end with this last 8. This value is rounded off, but there is no exact value for *e* because you can never find the *last* digit. Mathematicians call such numbers with no exact value, *irrational numbers*. The number represented by *e* appears in several electronics calculations, and is called the *natural number*, because it appears as a constant of nature. You will use *e* to calculate the voltage on a capacitor as it charges or discharges, for example.

Logarithms that use *e* for their base are called *natural logarithms*, or *Naperian logarithms*. This can be written as $\log_e$, but to more easily distinguish it from common logs, we usually abbreviate it ln. We define natural logs the same way we define common logs. If:

$M = e^y$, then
$\log_e(M) = \ln(M) = y$

The easiest way to find a natural log is with a scientific calculator. Enter the number whose ln you want to know, then press the "ln" button on the calculator. For example, ln (2) = 0.693 and ln (20) = 2.996. As you might expect, ln (*e*) = 1, ln (1) = 0 and ln (0) is undefined.

Inverse natural logs, or antilogs are also easy with a calculator. Just raise *e* to that power: $e^{2.996} = 20$.

Computers often work only with natural logarithms. Converting between common logarithms and natural logarithms is easy, however. If you want to find a common log, and know the natural log value, divide that by the natural log of 10.

$$\log(x) = \ln(x) / \ln(10) = \ln(x) / 2.3025851$$

$$\log(x) = 0.4342945 \ln(x)$$

If you know the common log, and want to find the natural log, divide that value by the common log of *e*.

$$\ln(x) = \log(x) / \log(e) = \log(x) / 0.4342945$$

$$\ln(x) = 2.3025851 \log(x)$$

DECIBELS

The bel (abbreviated B) is named after Alexander Graham Bell, who did much pioneering work with sound and the way our ears respond to sound. Our ears respond to sounds ranging from an intensity less than 10^{-16} W/cm^2 to intensities larger than 10^{-4} W/cm^2 (where we begin to experience pain). This is a range of more than 10^{12} times from the softest to the loudest sounds. Logarithms provide a convenient way to represent these values, because they compress this scale into a range of 12, rather than a range of a billion.

A bel is defined as the logarithm of a power ratio. It gives us a way to compare power levels with each other and with some reference power.

$$\text{bel} = \log\left(\frac{P_1}{P_0}\right)$$

where P_0 is the reference power, or the power you want to use for comparison and P_1 is the power you are comparing to the reference level.

While the bel was first defined in terms of sound power, to describe sound intensities, in electronics we often use it to compare electrical power levels. The decibel is one-tenth of a bel, and is abbreviated dB.

It takes 10 decibels to make 1 bel, so we can write an equation to find dB directly:

$$\text{dB} = 10\log\left(\frac{P_1}{P_0}\right)$$

How many decibels does the power increase if an amplifier takes a 1-W signal and boosts it to 50 W? Let P_1 be the 1-W signal in this example, since that is the starting point for the comparison.

$$\text{dB} = 10\log\left(\frac{50\text{ W}}{1\text{ W}}\right) = 10\log(50)$$

$$\text{dB} = 10(1.699) = 16.99\text{ dB}$$

The amplifier in this example has a gain of nearly 17 dB.

Sometimes when we are comparing signal levels in an electronic circuit, we know the voltage or current of the signal, but not the power. Of course we can always calculate the power, as long as we know the impedance of the circuit. We can take a shortcut to comparing the signal levels in decibels, however, *as long as the impedance is the same* in both circuits, or as long as the impedance of the circuit doesn't change when we change the voltage or current. Remember from Ohm's Law and the power equation that

$P = E^2 / R$ and $P = I^2 \times R$. So we can use E^2 or I^2 in place of power in the decibel equation, *as long as the impedance is the same* in both cases.

$$dB = 10 \log \left(\frac{E_1^2}{E_0^2} \right)$$

$$dB = 20 \log \left(\frac{E_1}{E_0} \right)$$

and

$$dB = 10 \log \left(\frac{I_1^2}{I_0^2} \right)$$

$$dB = 20 \log \left(\frac{I_1}{I_0} \right)$$

Here we have also illustrated another important property of logarithms. If the quantity inside the log expression has an exponent, you can move the exponent outside the log. In this case, we move the 2 from the squared terms out front, and multiply it times the 10 already there.

Sometimes there is confusion about whether the decibel was calculated using power, voltage or current. Since the current and voltage equations use 20 instead of 10 times the log term, some hams believe the "voltage" or "current" decibel is different than one calculated using power. This is not true, however. There is only one decibel definition, and that is ten times the log of a power ratio.

There are several power ratios that you should learn to recognize and remember the decibel values that go with them. These are the decibel values for a doubling of the power and for halving the power. Let's look at the effect of doubling the power first. It doesn't matter if we are going from 1 W to 2, 50 to 100 or 500 to 1000 W. In each case the new power is twice the starting power. To find the decibel increase multiply 10 times the log of 2:

$$dB = 10 \log (2)$$

$$dB = 10 \times 0.301 = 3.01$$

Anytime you double the power, it represents approximately a 3-dB increase in power.

What is the decibel change when you cut the power in half? Again, it doesn't matter if you are going from 1000 W to 500, 100 to 50 or 2 W to 1 W; the power ratio is still 0.5.

$$dB = 10 \log (0.5)$$

$$dB = 10 \times -0.301 = -3.01$$

A negative value indicates a decrease in power. Anytime you cut the power in half there is about a 3-dB decrease in power.

Table 4.7 shows the relationship between several common decibel values and the power change associated with those values. The current and voltage changes are also included, but these are only valid if the impedance is the same for both values.

Suppose you double the power, and then double it again? The final power is four times the starting power, so you can calculate the decibel increase using the equation given. You can also calculate the total power change "by inspection" because you know each time you double the power there is a 3-dB increase. In this example you have a 3-dB increase, plus a second 3-dB increase. If you add these two decibel values, you have a 6-dB total increase. If you double the power again, you have a 9-dB total increase. Doubling the power a fourth time gives a 12-dB total increase.

The same relationship is true of power decreases. Each time you cut the power in half you have a 3-dB decrease. Cutting the power in half and then in half again is a 6-dB decrease, and so on.

The addition and subtraction of decibel values is very important in electronics. Amplification factors, gains and losses of antennas, antenna feed lines and all kinds of circuits can simply be added when they are expressed in decibels.

Table 4.7
Some Common Decibel Values and Power-Ratio Equivalents

dB	P_2 / P_1	V_2 / V_1 or I_2 / I_1
–20	10^{-2}	0.1000
–10	0.1000	0.3162
(–6.0206)	(0.2500)	(0.5000)
–6	0.2512	0.5012
(–3.0103)	(0.5000)	(0.7071)
–3	0.5012	0.7079
–1	0.7943	0.8913
0	1.000	1.000
1	1.259	1.122
3	1.995	1.413
(3.0103)	(2.0000)	(1.4142)
6	3.981	1.995
(6.0206)	(4.0000)	(2.0000)
10	10.00	3.162
20	10^{2}	10.00

It is often convenient to compare a certain power level with some standard reference. For example, suppose you measured the signal coming into a receiver from an antenna and found the power to be 2×10^{-13} mW. As this signal goes through the receiver it increases and decreases in strength until it finally produces some sound in the receiver speaker or headphones. It is convenient to describe these signal levels in terms of decibels. A common reference power is 1 mW. The decibel value of a signal compared to 1 mW is specified as “dBm” to mean decibels compared to 1 mW. In our example, the signal strength at the receiver input is:

$$\text{dBm} = 10 \log \left(\frac{2 \times 10^{-13}\ \text{mW}}{1\ \text{mW}} \right)$$

$$\text{dBm} = 10 \log \left(2 \times 10^{-13} \right)$$

$$= 10 \times -12.7 = -127\ \text{dBm}$$

There are many other reference powers used, depending upon the circuits and power levels. If you use 1 W as the reference power, then you would specify dBW. Antenna power gains are often specified in relation to a dipole (dBd) or an isotropic radiator (dBi). Anytime you see another letter following the dB, you will know some reference power is being specified.

Integration and Differentiation

You don't have to be familiar with calculus to understand modern electronics. Sometimes it is helpful to be familiar with some calculus *terminology* to understand how a circuit works or what its function is, however.

INTEGRATION

When you read that a certain op-amp circuit is designed as an "integrator" it will be easier to understand what the circuit does if you know a simple definition of *integration*. Integration is the process of calculating the area under a curve plotted on a graph.

Area always implies certain boundaries, and you want to find how much space there is inside the boundaries. A square may be the simplest surface for which to find the area. If you know the length of a side you simply square that length to find the area. If you know the length and width of a rectangle you multiply these values to calculate area.

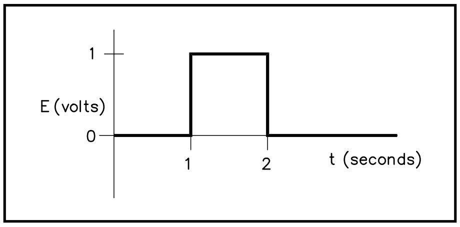

Fig 4.21 — This graph represents one cycle of a square wave. The text explains how to *integrate* this signal waveform, or find the area under the pulse.

Fig 4.21 shows a graph of a square-wave signal. If you want to *integrate* this signal, you have to find the *area* of the pulse. The scales on this graph represent voltage (on the Y axis) and time (on the X axis) so this isn't area in the most common sense, but we can perform a similar calculation. As you can imagine, if the pulse has a larger amplitude or a longer duration it will have a larger area.

Fig 4.22 illustrates a more difficult signal to integrate. Calculus methods can calculate this area from the equation that represents the curve, but we can make a reasonable approximation by drawing a series of rectangles and adding their areas. Integration is normally done over some range of values, such as x_1 and x_2 as shown on this graph. Part B shows that we can draw a series of rectangles so the *midpoint* of the side of each rectangle crosses the curve. Part of the rectangle corner lies above the curve, but there is a nearly equal space below the curve that is not included. If we draw more rectangles, with smaller widths, the approximation becomes better. The concept of integration is that you can make the interval smaller and smaller until it is no longer an approximation, but an exact value.

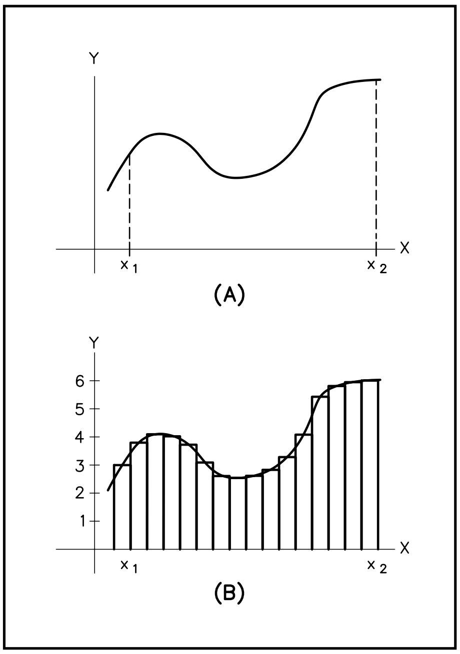

Fig 4.22 — This graph shows an irregular curve. If we want to know the area under the curve between x_1 and x_2, we can draw a series of rectangles and add their areas, as shown in Part B.

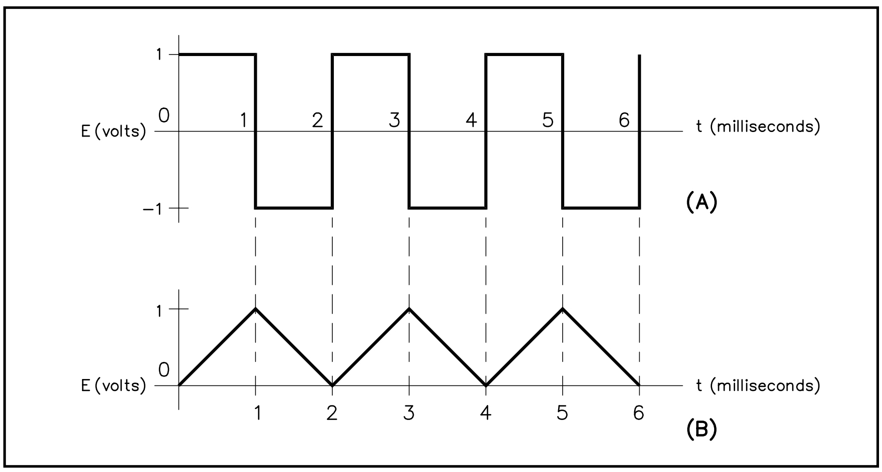

Fig 4.23 — Part A shows a series of square-wave pulses and Part B shows the output from an op-amp integrator with the square-wave input.

compares a series of square-wave pulses fed into an op-amp integrator and the output waveform from the integrator. In this example the integrator changes a square-wave signal into a triangle-wave signal. The integrated signal increases while the input pulse is positive, then decreases while the signal is negative.

DIFFERENTIATION

Differentiation is another calculus procedure that may be helpful. Integration and differentiation are opposite procedures. If you integrate a function and then differentiate the result, you get the original function back again. Likewise, if you differentiate a function and then integrate the result, you get the original function back.

While integration represents a summation of *area* values over some range, differentiation represents the *slope* of a line or curve at some specific point. The slope of a straight line is equal to the change in value along the x axis divided by the corresponding change in value along the y axis.

Look at the triangle waveform of Fig 4.23 B. While the voltage is increasing, this line has a constant slope, m, such that it satisfies the equation $y = mx + b$. Since the differentiation process represents the slope of the line, the *derivative* is a constant. When the waveform begins to decrease the slope suddenly changes to a new value, which is negative this time. The derivative is again a constant value, this time with a negative sign. If the graph in Fig 4.23B represents a signal waveform that is fed into a differentiator circuit, the waveform at A represents the output signal waveform!

We approximated the integration process for a curved-line graph by adding the areas of many small rectangles drawn to divide a curve into small segments. Similarly, we can approximate the differentiation process of a curved-line graph by finding the slope of a straight line drawn *tangent* to the curve. A tangent line touches the curve at a single point. The simplest way to show a tangent line is with a circle, as shown in **Fig 4.24**. The tangent line is perpendicular to (forms a 90° angle with) a radius line.

We can approximate the derivative of a curved-line graph at any point by drawing a tangent line at that point and calculating the slope of the line. We can even find the general trend of the derivative function by drawing a series of tangent lines at points along the line. By calculating the slope of each of those lines you can get some idea of how it is changing. By selecting points closer and closer together you will find a better and better approximation to the derivative

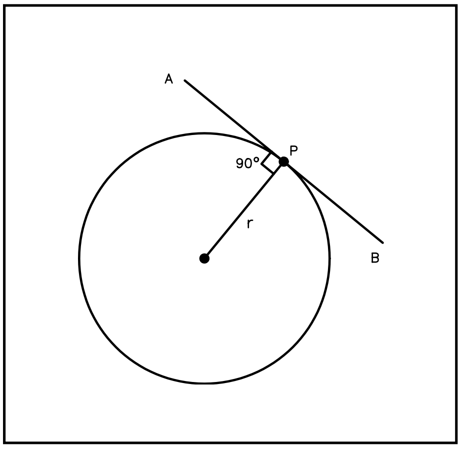

Fig 4.24 — This diagram illustrates the concept of a tangent line. Line AB is tangent to the circle at point P, and is perpendicular to the radius line, r.