Windowing filter coefficients   N1AL 6/9/2009

The filter coefficients for an FIR filter are just the impulse response of the filter. One method to compute the filter coefficients is to write an equation to describe the desired frequency response and then do an inverse Fourier transform to get the imulse response. However, an impulse response of finite length has ripples in the pass-band and stop-band. To reduce the ripples, the impulse response must be windowed.

As a demonstration of the technique, we define an ideal "brick-wall" filter in the frequency domain and run it through an inverse Fourier transform to get the impulse response (even though we already know that the answer is a sinc function). Then we window the result and do a forward Fourier transform on both the windowed and non-windowed versions so we can compare the amount of ripple.

$N := 128$ Number of filter coefficients $Fc := 0.125$ Filter cutoff frequency

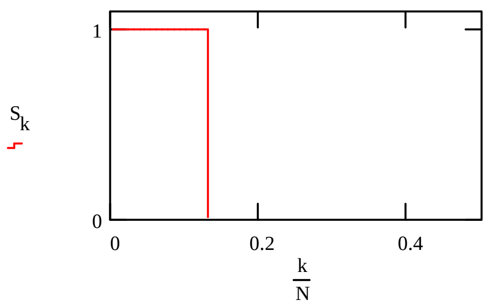
$n := 0 .. N-1$ Coefficient index $k := 0 .. \frac{N}{2}$ Frequency index

Define the frequency response of an ideal low-pass filter:

$S_k := 0 \qquad i := 0 .. N \cdot Fc \qquad S_i := 1.0$

The frequency response is first set to all zero.

Then the frequencies up to Fc = 0.125 are set to 1.0.



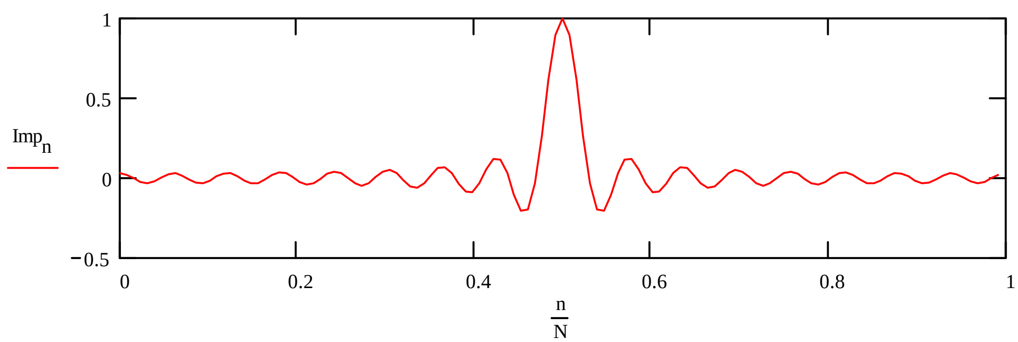
Now calculate the impulse response:

Get impulse response with inverse Fourier transform:

$I := \mathrm{IFFT}(S)$

Shift the coefficients to put the peak at the center:

$j := 0 .. \frac{N}{2} - 1 \qquad Imp_j := \frac{I_{j+\frac{N}{2}}}{I_0} \qquad Imp_{j+\frac{N}{2}} := \frac{I_j}{I_0}$



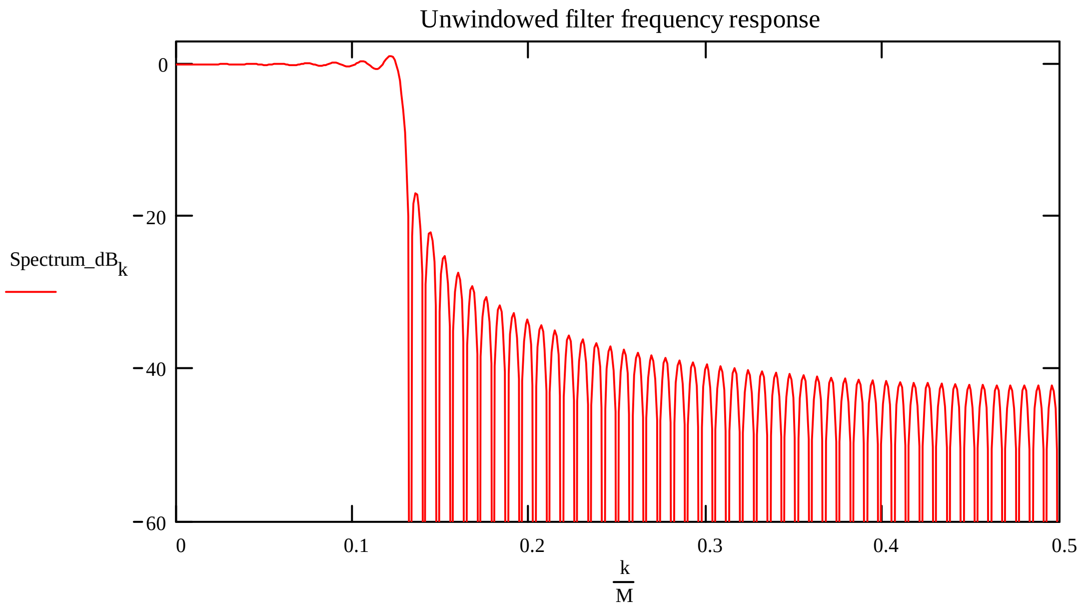
Extend the impulse response with zeros to simulate a non-repeating impulse response:

$M := 1024 \quad i := 0..M-1 \quad Impulse_i := 0 \quad Impulse_n := Imp_n$

Now let's see how we did.

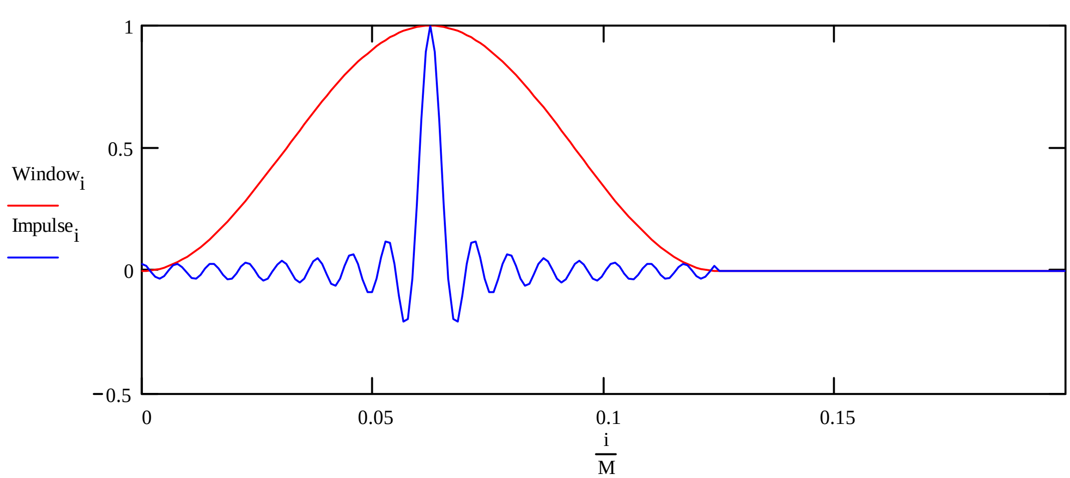
$Spectrum := FFT(Impulse) \quad k := 0..\frac{M}{2}$ Only half the coefficients are needed in the frequency domain since the spectrum is symmetrical.

$Spectrum\_dB_k := 20 \cdot \log\left(\left|\frac{Spectrum_k}{Spectrum_0}\right| + 10^{-7}\right)$ Convert to dB units



Now window the impulse response:

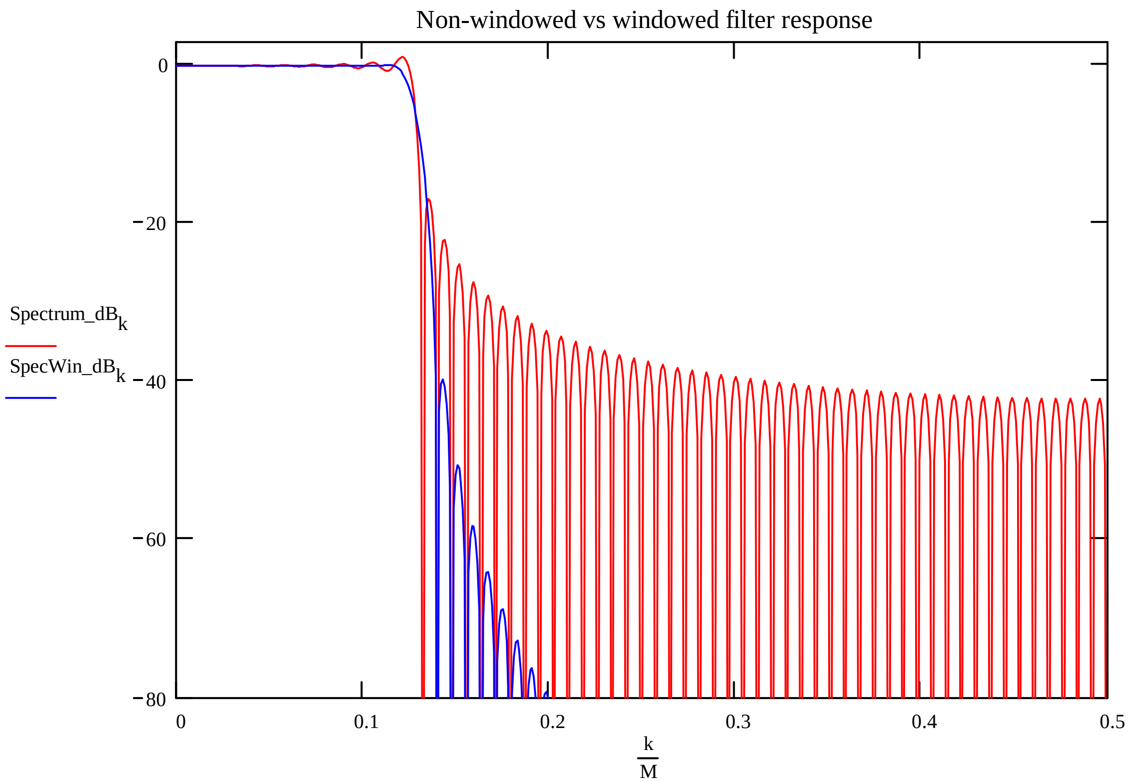
$Window_i := \frac{1}{2} \cdot \left[1 - \cos\left[2 \cdot \pi \cdot \left(\frac{1}{N} \cdot i\right)\right]\right]$ Hanning window $\quad j := N..M-1 \quad Window_j := 0$



$WindowedImpulse_i := Window_i \cdot Impulse_i$ Apply the window

$SpectrumWin := FFT(WindowedImpulse)$ $k := 0 .. \frac{M}{2}$

$$SpecWin\_dB_k := 20 \cdot \log\left(\left|\frac{SpectrumWin_k}{SpectrumWin_0}\right| + 0.0000001\right)$$



Ripple in both the passband and stopband is greatly reduced through windowing but the passband-to-stopband transition is not as sharp..