Reflections on the Smith Chart

Although most radio amateurs have seen the Smith Chart, it is often regarded with trepidation. It is supposed to be complicated and subtle. However, the chart is extremely useful in circuit analysis, especially when transmission lines are involved. The Smith Chart is not limited to transmission-line and antenna problems.

The basis for the chart is Eq 4 in the main text relating reflection coefficient to a terminating impedance. Eq 4 is repeated here:

$$\rho = \frac{Z - Z_0}{Z + Z_0} \quad (1)$$

where Z_0 is the characteristic impedance of the chart, and $Z = R + jX$ is a complex terminating impedance. Z might be the feed-point impedance of an antenna connected to a Z_0 transmission line.

It is useful to define a normalized impedance $z = Z/Z_0$. The normalized resistance and reactance become $r = R/Z_0$ and $x = X/Z_0$. Inserting these into Eq 1 yields:

$$\rho = \frac{z - 1}{z + 1} \quad (2)$$

where r and z are both complex, each having a magnitude and a phase when expressed in polar coordinates, or a real and an imaginary part in XY coordinates.

Eq 1 and 2 have some interesting

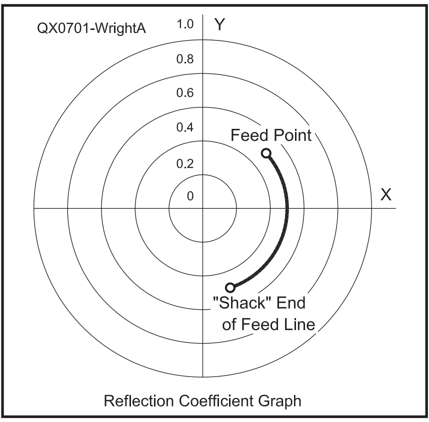

Fig A—Plot of polar reflection coefficient. Circles represent contours of constant ρ. The starting "feed point" value, 0.5 at +45°, represents an antenna impedance of 69.1 + *j* 65.1 Ω with Z_0 = 50 Ω. The arc represents a 15-ft section of 50-Ω, VF 0.66 transmission line at 7 MHz, yielding a shack ρ of 0.5 at –71.3°. The shack z is calculated as 40.3 – *j* 50.9 Ω.

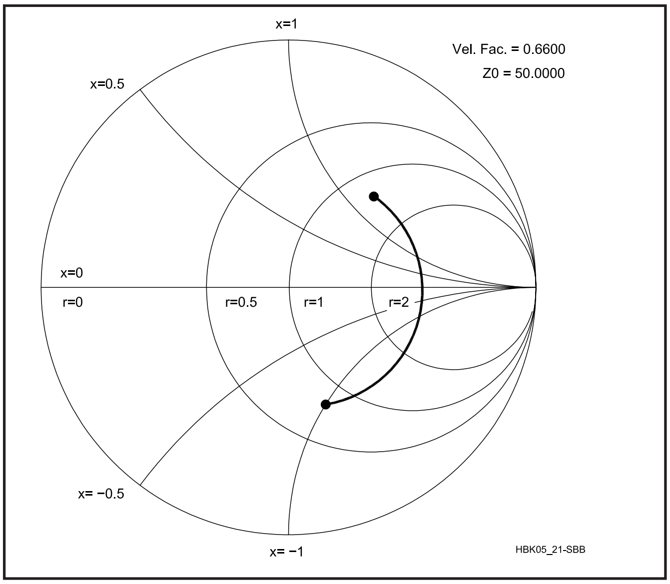

Fig B—This plot shows a Smith Chart. The circles now represent contours of constant normalized resistance or reactance. Note the arc with the markers: This illustrates the same antenna and line used in the previous figure. The plot is the same on the two charts; only the scale details have changed.

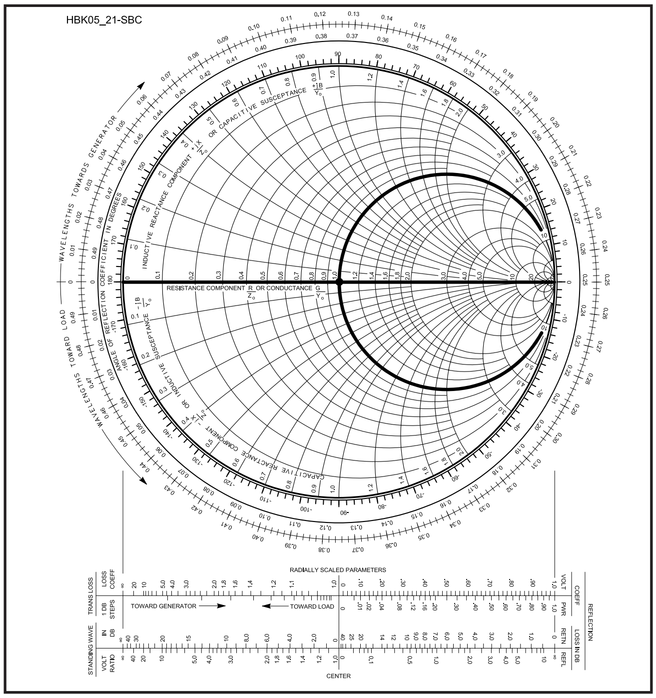

Fig C—The Smith Chart shown in Fig B was computer generated. A much more detailed plot is presented here; this is the chart form used by Smith, suitable for graphic applications. This chart is used with the permission of Analog Instruments.

and useful properties, characteristics that make them physically significant:

- Even though the components of z (and Z) may take on values that are very large, the reflection coefficient ρ, is restricted to always having a magnitude between zero and one if z has a real part, r, that is positive.
- If all possible values for ρ are examined and plotted in polar coordinates, they will lie within a circle with a radius of one. This is termed *the unit circle*. A plot is shown in **Fig A**.
- An impedance that is perfectly matched to Z_0, the characteristic value for the chart, will produce a ρ at the center of the unit circle.
- Real Z values, ones that have no reactance, "map" onto a horizontal line that divides the top from the bottom of the unit circle. By convention, a polar variable with an angle of zero is on the x axis, to the right of the origin.
- Impedances with a reactive part produce ρ values away from the dividing line. Inductive impedances with the imaginary part greater than zero appear in the upper half of the chart, while capacitive impedances appear in the lower half.
- Perhaps the most interesting and exciting property of the reflection coefficient is the way it describes the impedance-transforming properties of a transmission line, presented in closed mathematical form in the main text as Eq 11.

Neglecting loss effects, a transmission line of electrical length θ will transform a normalized impedance represented by ρ to another with the same magnitude and a new angle that differs from the original by –2θ. This rotation is clockwise.

Clearly, the reflection coefficient is more than an intermediate step in a mathematical development. It is a useful, alternative description of complex impedance. However, our interest is still focused on impedance; we want to know, for example, what the final z is after transformation with a transmission line. This is the problem that Phillip Smith solved in creating the Smith Chart. Smith observed that the unit circle, a graph of reflection coefficient, could be labeled with lines representing *normalized impedance*. A Smith Chart is shown in **Fig B**. All of the lines on the chart are complete or partial circles representing a line of constant normalized resistance and reactance.

How might we use the Smith Chart? A classic application relates antenna feed-point impedance to the impedance seen at the end of the "shack" end of the line. Assume that the antenna impedance is known, $Z_a = R_a + j X_a$. This complex value is converted to normalized impedance by dividing R_a and X_a by Z_0 to yield $r_a + j x_a$, and is plotted on the chart. A compass is then used to draw an arc of a circle centered at the origin of the chart. The arc starts at the normalized antenna impedance and proceeds in a clockwise direction for 2θ°, where θ is the electrical degrees, derived from the physical length and velocity factor of the transmission line. The end of the arc represents the normalized impedance at the end of the line in the shack; it is denormalized by multiplying the real and imaginary parts by Z_0.

Antenna feed-point Z can also be inferred from an impedance measurement at the shack end of the line. A similar procedure is followed. The only difference is that rotation is now in a counterclockwise direction. The Smith Chart is much more powerful than depicted in this brief summary. A detailed treatment is given by Phillip H. Smith in his classic book: *Electronic Applications of the Smith Chart* (McGraw-Hill, 1969). I also recommend his article "Transmission Line Calculator" in Jan 1939 *Electronics*. Joseph White presented a wonderful summary of the chart in a short but outstanding paper: "The Smith Chart: An Endangered Species?" Nov 1979 *Microwave Journal*. *—Wes Hayward, W7ZOI*