Fifth-order sine approximation N1AL 5/28/2009

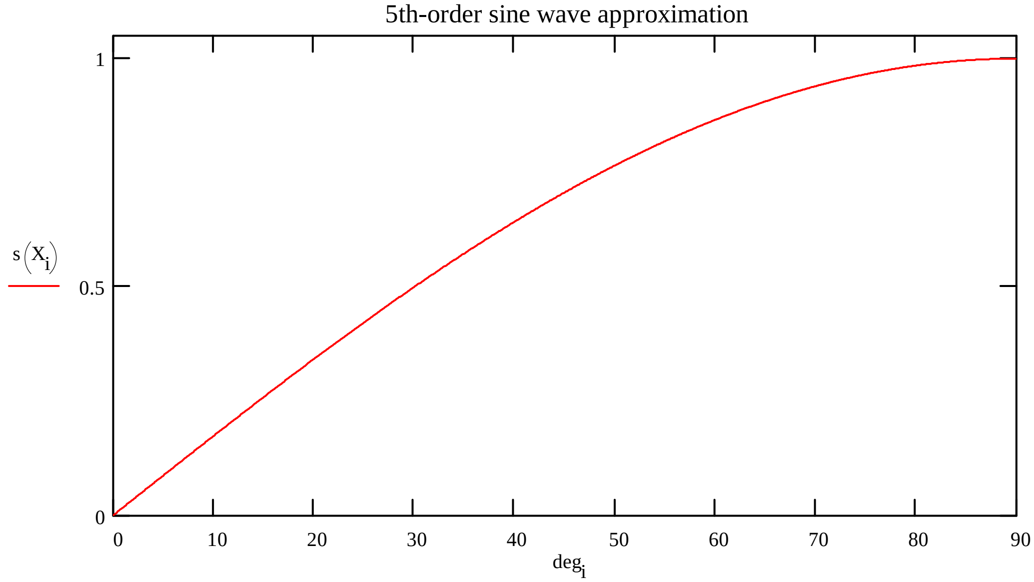
It is possible to obtain quite a good approximation to a sine wave between 0 and 90 degrees using a fifth-order curve fit. Although this only works for the first quadrant of one complete 360-degree cycle, the other quadrants are all reversed and/or inverted versions of the first, so with additional hardware or software the other three quadrants can be calculated from the first.

$$C := \begin{bmatrix} 0 \\ 3.14062500 \\ 0.02026367 \\ -5.32519600 \\ 0.5446778 \\ 1.800293 \end{bmatrix}$$

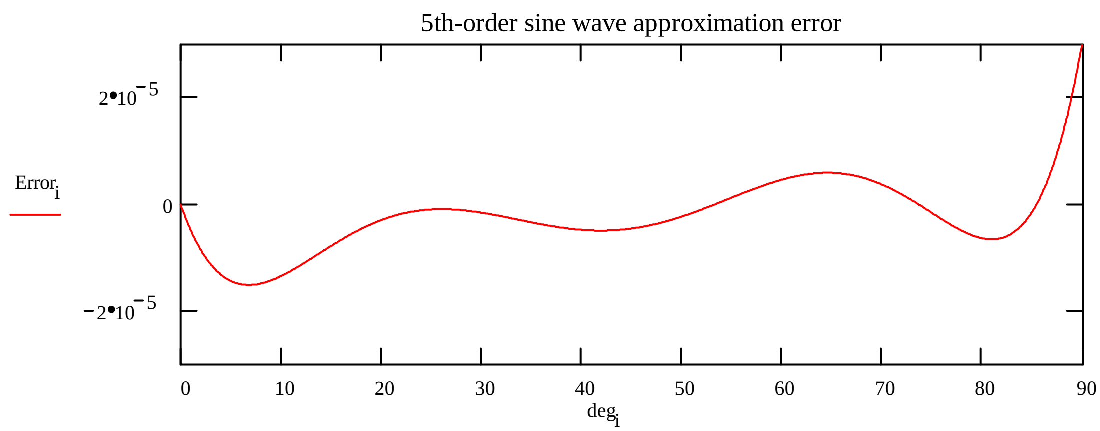
The coefficients are from the Analog Devices ADSP2100 Users Guide

$s(x) := C_1 \cdot x + C_2 \cdot x^2 + C_3 \cdot x^3 + C_4 \cdot x^4 + C_5 \cdot x^5$ Fifth order approximation.

$size := 1024$ $i := 0 .. size$ $X_i := \frac{i}{size \cdot 2}$ X=1.0 corresponds to 180 degrees. $deg_i := X_i \cdot 180$



$Error_i := s(X_i) - \sin(\pi \cdot X_i)$



Create the four quadrants of one cycle of the sine wave:

$sine_i := s(X_i)$ $\quad sine_{i+size} := sine_{size-i}$ $\quad i := 0..\ 2 \cdot size - 1$ $\quad sine_{2 \cdot size + i} := -sine_i$

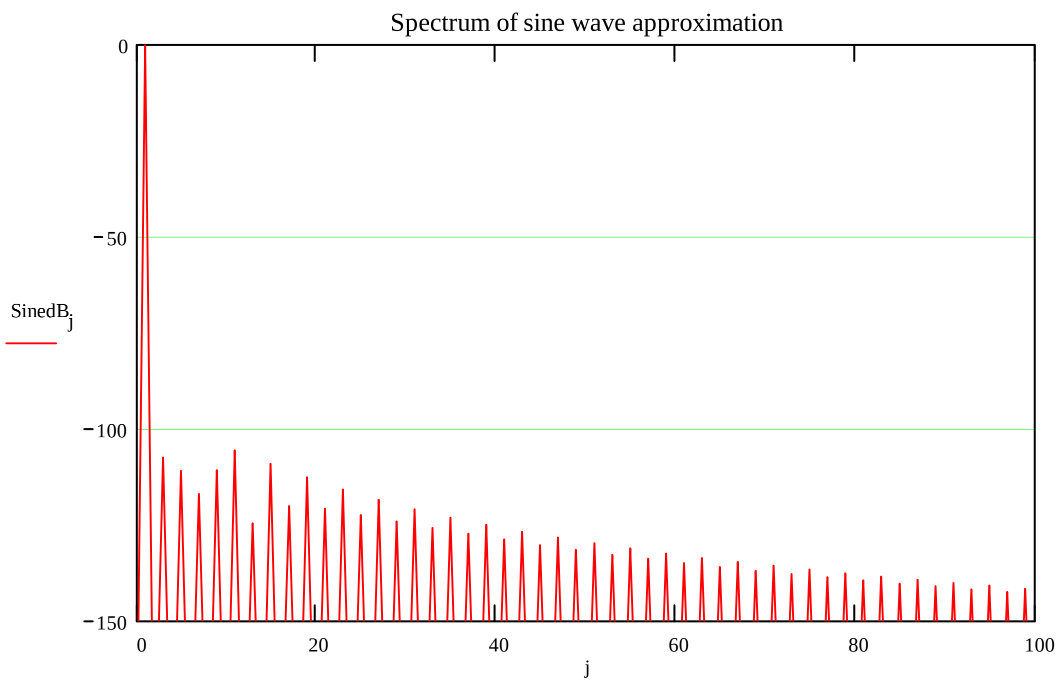
$Sine := 2 \cdot FFT(sine)$ Compute the frequency spectrum

$j := 0..\ 2 \cdot size$

There are 4*size points in the time sequence but only 2*size points in the frequency spectrum. That is because with a real sequence, the frequency spectrum is symmetrical, so only half the points are needed.

$SinedB_j := 20 \cdot \log\left(\left| Sine_j \right| + 10^{-10} \right)$

Convert the frequency spectrum to dB.
(The 10^{-10} is to avoid taking the log of zero.)



All harmonics are over 100 dB down. Note that only odd harmonics are present