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Title: Experiment #32--Thevenin Equivalents
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Experiment #32—Thevenin Equivalents

You can't buy a Thevenin Equivalent at the local electronics emporium, but it exists behind every pair of terminals—a fairly simple concept that you can use in many circumstances, a part of every electronic designer's tool set.

Terms to Learn

- *Duality*—the symmetry of opposite quantities, such as voltage and current or resistance and conductance, that allow electrical circuits to be described in terms of either.
- *Equivalent*—a circuit that behaves exactly the same as the original

Introduction

You've already encountered one aspect of Thevenin Equivalents by measuring the output impedance of the common-base amplifier circuit in Experiment 28. (All previous experiments are available at **www.arrl.org/tis/info/HTML/Hands-On-Radio**.) The output of the amplifier was acting like a perfect signal source in series with the measured resistance. Output impedance is just one instance of the more general idea of source impedance.

There are two kinds of sources that supply electrical energy in the form of voltage or current. An *ideal source* is one that can supply any amount of energy required while maintaining its rated voltage or current. For example, the ideal voltage source, shown in Figure 1, can satisfy Ohm's Law (I = V/R) through the attached resistor, whether the resistor is 1 MΩ or 1 µΩ.

An ideal source would be a handy thing; it doesn't really exist but needs only to be approximated. If I am trying to build an op-amp filter, my voltage source (that is, the power supply) may only have to supply a few mA to act as close to ideal as I need. To build a power grid, I need the Grand Coulee Dam's generators instead!

An actual voltage source can only supply a limited amount of power and still have a constant voltage or current. If I draw current from a battery, with a light bulb, for example, in Figure 1, the output voltage will drop. This voltage drop is caused by the battery's internal or source impedance. I can model the battery as an actual voltage source—an ideal source (V_S) in series with the internal impedance (R_S). The voltage at the battery's terminals, V_L, is reduced as the output current flows through the internal resistance:

$$I = V_S / (R_S + R_L) \qquad \text{[Eq 1]}$$

$$V_L = V_S - I\,R_S = (V_S - V_S\,R_S)/ (R_S + R_L)$$
$$= V_S\,[1 - R_S / (R_S + R_L)] \qquad \text{[Eq 2]}$$

Equation 1 shows that for an actual voltage source as R_L increases, the output voltage will drop and vice versa. If R_L is infinite (no load), the output or terminal voltage will be the same as that of the ideal source, since no current flows through the internal impedance. If R_L is zero (a short circuit), then the current is limited to V_S / R_S.

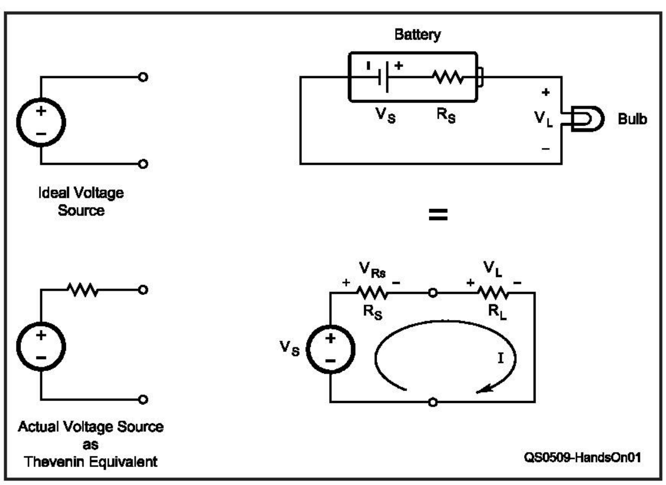

Figure 1—The Thevenin equivalent circuit is constructed from an ideal voltage source in series with a resistance.

Thevenin's Insight

From the Wikipedia (**www.wikipedia.com**): "Leon Charles Thevenin (1857-1926) was a French telegraph engineer who extended Ohm's law to the analysis of complex electrical circuits." Thevenin's main legacy is a simple but powerful statement: For a circuit made up of any combination of voltage sources and resistors, the behavior at a pair of terminals to that circuit can be completely replicated by a circuit consisting of a single ideal voltage source, called the Thevenin voltage (V_{TH}), in series with a single resistance, called the Thevenin resistance (R_{TH}). That is Thevenin's Theorem, and the source and resistance comprise the Thevenin Equivalent in Figure 1.

That's a pretty powerful idea! If the original circuit, no matter how complicated, was out of your sight with only a pair of terminals visible, you couldn't tell whether the circuit was the complicated one or its Thevenin equivalent. All you know about it is the behavior at its external terminals. Thevenin's Theorem also works for ac signals and circuits, using ac signal sources and impedances. In general, the theorem is true as long as all of the sources and components respond linearly to voltage and current—no diodes or relays, for example.

Equivalence, More or Less

When you're designing or analyzing a circuit, it's handy to be able to replace parts of it with simplified circuit bits that act the same but are easier to analyze. These are equivalent circuits. For example, if I'm working on an amplifier circuit, I need to know the characteristics of what is connected to its input terminals. I don't really want to deal with the whole circuit of the signal source, so I replace it with its Thevenin equivalent. The Thevenin resistance is what you measured as output impedance in Experiment #28.

An equivalent circuit doesn't need to be a Thevenin equivalent. If you can replace a circuit of resistors in parallel with a

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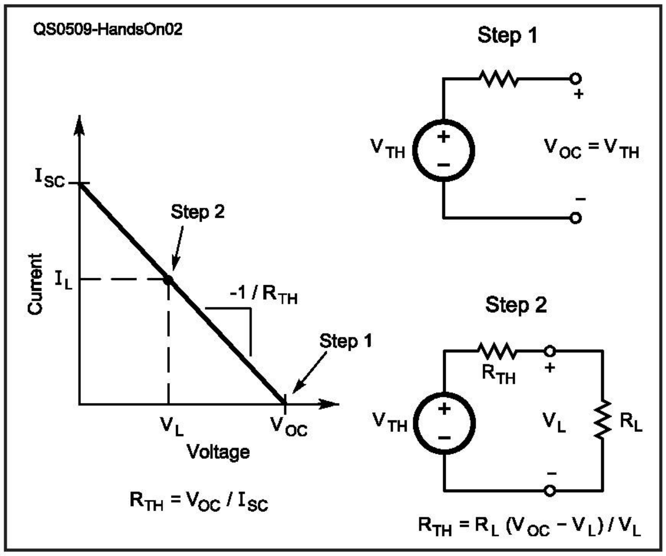

Figure 2—The values of the Thevenin equivalent circuit can be determined by simple measurements of voltage and current.

different circuit that has resistors in series, the new circuit is the original's series equivalent. (Going the other way, there's also a parallel equivalent.) A circuit that responds the same only at dc is the dc equivalent. All "equivalent circuit" means is that you can replace a circuit with its equivalent without affecting the external behavior.

Measuring a Thevenin Equivalent

Let's figure out the Thevenin equivalent circuit for a battery by determining V_{TH} and R_{TH}. A battery makes a pretty good voltage source, but it still has some internal resistance. How can you figure out what that internal resistance is? We'll use almost the same technique as was used to measure output impedance. Figure 2 illustrates the two steps and how the measurements allow you to determine both V_{TH} and R_{TH}.

- Obtain a 1.5 V AAA or 9 V battery. Do *not* use a larger battery because they can supply enough current to cause a burn or damage themselves.
- The load will be a 100 Ω, 1 W resistor; either buy one or make it from several lower-power resistors. Measure the exact value with your voltmeter. It doesn't have to be exactly 100 Ω; anything from 25 to 100 Ω will do. Don't use a light bulb; the resistance changes dramatically as it heats up.
- With no load connected at all, measure the open-circuit voltage of the battery. Since no current flows in R_{TH}, this is the same voltage as V_{TH}.
- Connect your voltmeter to the resistor leads, and using pliers or tweezers apply the resistor leads directly to the battery terminals without using clip leads or other connectors. Don't hold the resistor or its leads in your fingers, as it may get hot!
- The voltage will drop by some amount to V_L. For fresh batteries the voltage may not drop much. You can increase the voltage drop by lowering R_L, taking care to provide enough power dissipation. ($P = V^2 / R$)
- Use the equation in Figure 2 to determine R_{TH}. Congratulations: You just determined the Thevenin equivalent of your battery! Whatever is inside your battery—more batteries, chemicals, or a tiny hamster on a wheel—can be replaced by an ideal voltage source, V_{TH}, and series resistance, R_{TH}.
- Experiment with fresh and depleted batteries of the same type to see how their Thevenin circuit changes with energy level. Try different types of batteries, as well, taking care to avoid excessive heat dissipation in the load resistor.

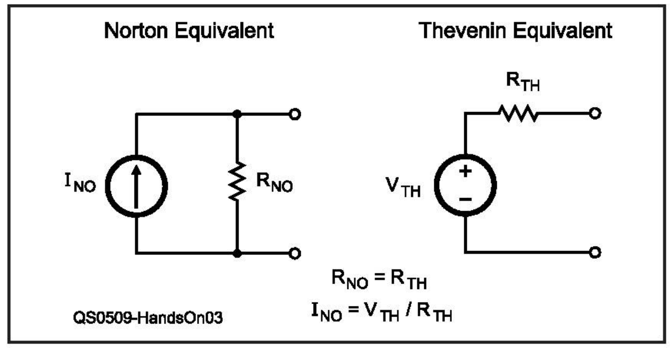

Figure 3—The Norton Equivalent and Thevenin Equivalent circuits exhibit exactly the same behavior at their outputs and can replace each other directly.

The graph in Figure 2 illustrates the process of determining V_{TH} and R_{TH}. With no current drawn from the source, the open-circuit and Thevenin voltages are equivalent. If you short-circuit the output terminals, the current is limited to I_{SC} by R_{TH}. The negative reciprocal of the slope of the line between V_{OC} and I_{SC} is equal to R_{TH}. It's often not advisable to use a short circuit because of the potential damage to the circuit being tested or the circuit tester. Use an intermediate point with a safe current and determine R_{TH} from the slope of the line between that point and the open-circuit point.

Norton Equivalent

There is another way to create an equivalent circuit using an ideal current source and a resistance in parallel; this is called a Norton Equivalent, shown in Figure 3. The ideal current source supplies a fixed amount of current to whatever load is connected to its terminals. Surprisingly, R_{TH} has the same value in both the Norton and Thevenin equivalent circuits!

If you short-circuit a Thevenin circuit, $I_{SC} = V_{TH} / R_{TH}$. If the output is open-circuited, the voltage across the terminals is $I_{SC} \times R_{TH} = V_{OC} = V_{TH}$. Construct the Norton Equivalent by replacing the Thevenin voltage source with a current source of value I_{SC} and place R_{TH} across it. You get exactly the same graph of current and voltage at the output terminals—the definition of an equivalent circuit.

Dual Challenges

Thevenin and Norton circuits are a window into the world of *duality* in electronics where electrical behavior can be defined in terms of current or voltage. Other dual quantities include resistance and conductance, impedance and admittance, series and parallel, and node and mesh (from Experiment #29). The use of one parameter or the other is just a matter of convenience and always leads to the same answer for electrical energy and power.

Shopping List

- AAA or 9 V battery
- 100 Ω, 1 W resistor

Suggested Reading

Thevenin and Norton equivalents are discussed with examples on pages 4.5 and 4.6 of *The 2005 ARRL Handbook*. A more extensive discussion is available on-line at **www.allaboutcircuits.com/vol_1/chpt_10/7.html**. That Web page also has extensive links to other aspects of electrical circuits.

Next Month

In October, we'll get to know the magnetic personality of one of the electronic world's most common components—the transformer.