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Simplified Design of Impedance-Matching Networks

In Three Parts
Part I—Basic Principles and the L Network

BY GEORGE GRAMMER, W1DF

• Breaking down the design of matching circuits into a step-by-step process results in a method that is not only as simple to understand as anything of this nature ever can be, but in which the desired numerical results are obtained through the use of the most elementary type of arithmetic. The second part of the article will take up the design of pi and T networks and the third will discuss some special applications and practical features.

THE GENERAL PROBLEM of transferring r.f. power from one device to another is one of "matching impedances." This is a term for the process of transforming the resistance of the device that receives the power into a value which the device that furnishes the power wants to have as a load. The power-receiving device may be, for example, a flat 52-ohm line. The power-furnishing device may be the final amplifier tube in a transmitter, and may require a load resistance of say 2000 ohms for delivering the desired amount of power at good efficiency. To transfer the power from the tube to the line, the amplifier output circuit must transform the 52-ohm actual load into a 2000-ohm load as seen by the tube.

The design of such matching circuits or "networks" is surprisingly simple, provided it is broken down into a series of steps. To use the process intelligently, it is necessary to understand the circuit action that gives the resistance transformation, but this is not difficult if the meaning and behavior of reactance are appreciated.[1] Reactance is the key to the operation of practically all r.f. circuits, and without an understanding of it there is little hope of being able to design such circuits.

Resistance and Impedance

The resistances to be matched are seldom actual resistors. The term resistance is used here in its broader interpretation as the voltage-to-current ratio at which power is consumed or transferred. Thus a resonant antenna has a "resistance" of 70 ohms because the current in amperes that flows into its terminals is 1/70 of the number of volts applied to the terminals. A flat 52-ohm line has a resistance of 52 ohms because the current in amperes is equal to 1/52 of the volts applied to the line. Neither the antenna nor the line actually *consumes* power; each simply passes it on to something else. For the purpose of circuit design it is convenient to substitute the resistance symbol for these and similar devices, because their behavior conforms to that of actual resistances.

The term "impedance" is used in a comparable sense. It too is a voltage-to-current ratio. It is a more general term than resistance because it implies that all of the power supplied may not be consumed or passed on, but a certain proportion of it may be returned to the source during some part of the a.c. cycle. When this happens the actual device, be it antenna, transmission line or whatnot, can be represented by a combination of resistance and reactance. The resistive part represents the voltage-to-current ratio at which power is either consumed or passed on; the reactive part the voltage-to-current ratio at which the power is returned to the source.[2]

Determining the values of resistance to be matched is often a more difficult problem than designing the circuit to match them. This question can in no case be ignored, but in the present discussion we shall lay it aside and deal with the subject of matching as such.

Equivalence of Series and Parallel Circuits

The basis for many kinds of impedance matching is the fact that for any circuit consisting of resistance and reactance in series there can be found a circuit consisting of resistance and reactance in parallel that will have exactly the same impedance and phase angle.

Thus the series and parallel circuits of Fig. 1 are exactly equivalent if, when a voltage of fixed magnitude and frequency is applied to either circuit, the same value of current results in both cases, and if the phase between current and voltage is also the same. If the two circuits were concealed in separate boxes, there would be no way to tell which of them actually was connected to the voltage source. This means that a simple series combination of resistance and reactance

[1] The subject of reactance is covered in sufficient detail for this purpose in the section on alternating currents in Chapter 2 of the *Handbook*.

[2] The energy that is stored in the electric or magnetic fields of the reactive elements during part of the a.c. cycle is taken from the fields and restored to the circuit — i.e., the source of power — during a subsequent part of the cycle. This "reactive power" is not consumed anywhere in the system, but simply is handed back and forth between the power source and the reactive elements.

can be lifted out of a more complex circuit and its parallel equivalent substituted for it without in any way affecting the over-all operation of the circuit. It is necessary to specify that the frequency remain fixed, because the reactance values change with a change in frequency.[3]

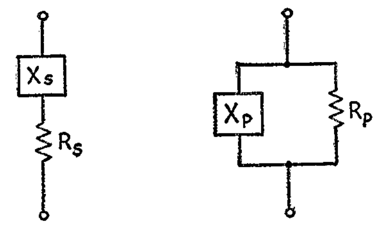

Fig. 1 — Series and parallel circuits containing resistance and reactance. By proper choice of constants, the two circuits will behave identically; i.e., the current and phase angle will be the same in both for the same impressed voltage.

In Fig. 1 the reactances are shown simply as blocks, since the same principles apply whether the reactance is inductive or capacitive. However, if the series reactance, X_S, is inductive the parallel reactance, X_P, in the equivalent parallel circuit also will be inductive, and vice versa. The reactances in such equivalent circuits always are of the same kind. Their values, however, are not identical: that is, X_S is not equal to X_P, and R_S is not equal to R_P. R_S will always be smaller than R_P, and X_S will always be smaller than X_P.

In determining the actual R and X values in the equivalent circuits, it is convenient to introduce the quantity Q. It has the same meaning as the one we ordinarily associate with that letter. That is, in the series circuit

$$Q = \frac{X_S}{R_S} \qquad (1A)$$

and in the parallel circuit

$$Q = \frac{R_P}{X_P} \qquad (1B)$$

When series and parallel circuits are equivalent, Q has the same value in both.[4]

From ordinary a.c. circuit theory it can be shown that a parallel circuit is equivalent to a given series circuit when

$$R_P = R_S(Q^2 + 1) \qquad (2A)$$

$$\text{and } X_P = \frac{R_P}{Q} \qquad (2B)$$

[3] Also, in many practical cases such a substitution might entail a change in accessory circuit details, such as direct-current feed. Obviously, d.c. would not flow through a series capacitor, although it would flow through a resistor in parallel with a capacitor. The discussion here is confined to the alternating-current operation of the circuit.

[4] It is necessary to keep in mind that the Q under consideration is the "operating" Q of the circuit, not the Q of a component, such as a coil. The latter Q is determined by the inherent resistance of the component. In most practical cases the power loss in a component (as represented by its internal resistance) will be very small compared with the power used in the load, so the component resistance can be neglected. The circuit or operating Q is therefore based on the load resistance.

while a series circuit is equivalent to a given parallel circuit when

$$R_S = \frac{R_P}{Q^2 + 1} \qquad (3A)$$

$$\text{and } X_S = QR_S \qquad (3B)$$

When the values of resistance and reactance satisfy these equations the two circuits will have exactly the same impedance and phase angle at the frequency considered.

The significant point in all this is that when the equivalence is achieved, the resistance values are not identical. Herein lies the clue to the matching properties.

Matching by Means of Reactance

Going back to the illustration mentioned earlier, of a 52-ohm load that had to be transformed into 2000 ohms so a tube could deliver its power output to a transmission line, let us assume that 52 ohms (the smaller of the two resistances) corresponds with R_S in Fig. 1. From the preceding discussion we may infer that if a suitable value of reactance, X_S, is added in series with the 52-ohm resistance we can come out with a circuit that is equivalent to a resistance of 2000 ohms in parallel with some value of reactance X_P.

Equation 2A can be rearranged to read

$$\frac{R_P}{R_S} = Q^2 + 1, \qquad (4)$$

which says that the ratio of the two resistances, R_P and R_S, corresponds with a specific value of Q, which is

$$Q = \sqrt{\frac{R_P}{R_S} - 1}. \qquad (5)$$

This is the relationship we need for matching purposes. In the illustration, R_P/R_S is 2000/52, which is equal to 38.4. Hence to transform 52 ohms into 2000 ohms Q must be equal to

$$Q = \sqrt{38.4 - 1} = \sqrt{37.4} = 6.1$$

The required value of series reactance X_S is found from Equation 3B, and is

$$X_S = 6.1 \times 52 = 318 \text{ ohms}$$

Thus a reactance of 318 ohms in series with the 52-ohm resistive load will make the circuit "look like" a resistance of 2000 ohms (which is what we want) in parallel with a reactance X_P (which we do not want particularly), the value of which is found from Equation 2B:

$$X_P = \frac{2000}{6.1} = 328 \text{ ohms.}$$

The equivalence is shown in Fig. 2. In this figure

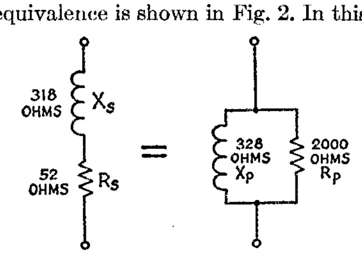

Fig. 2 — An example of series and parallel circuits that are equivalent.

it is assumed that inductive reactance is used, but capacitive reactance of the same numerical value would do equally well.

Since we originally wanted only a *resistive* load of 2000 ohms for the tube, something has to be done about the 328-ohm reactance in parallel with it. Before taking up that question, it may be observed that Fig. 3 gives in graph form the values of Q required for matching any two resistances having a ratio from 1 to 1000. For ratios above 100, the error in dropping the numeral 1 from Equation 5 will be negligible, so the relationship becomes

$$Q = \sqrt{\frac{R_P}{R_S}} \qquad (6)$$

Use of Fig. 3 obviates the necessity for taking the square root called for in Equations 5 or 6. Equations 2B and 3B call for nothing more than simple multiplication and division. It could hardly be said that the process of finding the proper value of reactance is complicated or difficult.

Circuit Action

The physical process by which the resistance transformation takes place poses no mystery. Adding reactance in series with resistance raises the impedance of the circuit, and the total impedance can be increased to any desired value by this method. On the other hand, if we want to develop a given amount of power in R_S, we must put a fixed amount of current through it regardless of the reactance in series. Hence, as the ratio of X_S to R_P (that is, Q) is increased by adding more and more reactance at X_S, more and more voltage is needed to force the same current through the circuit and thereby maintain the same power in R_S.

Suppose, in the illustration, that we want 52 watts in the 52-ohm resistance. In the resistance alone, this would require 52 volts and the current would be 1 ampere. If reactance is now added in series, the voltage must be increased to keep the current at 1 ampere. Eventually, as X_S is made larger, we reach the value of 318 ohms and find that the impedance of the circuit is

$$Z = \sqrt{(318)^2 + (52)^2} = 322 \text{ ohms}$$

To put 1 ampere through this circuit requires 322 volts.

Although the product of 322 volts and 1 ampere is 322 volt-amperes, the actual *power* is still 52 watts, because the reactance does not use up power. Nevertheless, the 52 watts is now being supplied to the *circuit* at 322 volts instead of 52 volts. If a circuit consumes 52 watts at 322 volts, Ohm's Law tells us that the resistance of that circuit should be

$$R = \frac{E^2}{P} = \frac{(322)^2}{52} = 2000 \text{ ohms}$$

On the other hand, a 2000-ohm resistor across a 322-volt source should take only 322/2000 or 0.161 ampere, whereas the actual current through the circuit is 1 ampere. The "excess" current is the current flowing through the parallel reactance, X_P. The current in this reactance has just the right value to make the total current become 1 ampere when combined with the 0.161 ampere flowing in R_P.

The L Section

Demanding that a source of power furnish 322 volt-amperes in order to deliver 52 useful watts would hardly be sporting, so something needs to be done to circumvent this aspect of the otherwise beneficial effect of the series reactance. The solution, which is quite simple, is variously called "power-factor correction," "reactance cancellation," or just "tuning to resonance."

It will be recalled that in a pure capacitance the current is a quarter cycle *ahead* of the applied voltage, while in a pure inductance the current is a quarter cycle *behind* the applied voltage. These currents are numerically equal when the reactances are numerically equal and the same voltage is applied to both. If we place two such reactances across a source of voltage, the leading current through the capacitance just balances the lagging current through the inductance, and if the two reactances are the only circuit elements connected to the voltage source that source

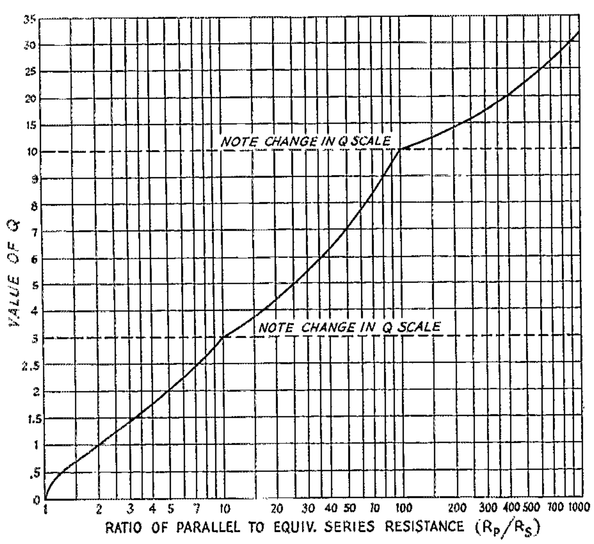

Fig. 3 — Q required for matching resistances having ratios (R_P/R_S) from 1 to 1000.

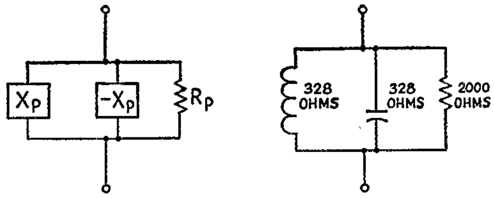

Fig. 4 — Reactance cancellation in the equivalent parallel circuit.

does not even know the reactances exist. In other words, no current flows out of the source even though large currents may be flowing in the capacitance and inductance.[5]

This type of circuit action is just what is needed for getting rid of the unnecessary volt-amperes. By placing a reactance having the same value as X_P, but of the opposite kind, in parallel with X_P all the reactance is effectively eliminated from the equivalent parallel circuit and the resistance alone is left. This is indicated in Fig. 4 by using a minus sign to show that the reactance is of the opposite kind. In the illustration of Fig. 2, where we have 328 ohms of inductive reactance in parallel with 2000 ohms of resistance, it is necessary to add a capacitive reactance of 328 ohms in parallel as shown at the right in Fig. 4. This cancels the inductive reactance and leaves just the 2000-ohm resistance.

Of course the actual circuit we began with is the one at the left in Fig. 2. The parallel equivalent at the right in that figure is just that — an equivalent. It is not the physical circuit even though it exhibits exactly the same impedance and phase angle as the series circuit. So when the compensating reactance, $-X_P$, is added in parallel the resulting physical circuit is as shown in Fig. 5. R_P is now shown with an arrow to indicate that it is the resistance that a power source connected to the terminals would "see." The physical configuration of the illustrative circuit is also shown in Fig. 5. If a flat 52-ohm line were connected to replace the 52-ohm resistor, a power tube connected to the circuit terminals would see the 2000-ohm load it wants, and the power output would be delivered to the transmission line without loss.

This circuit is the "L section," and it develops quite naturally and easily out of the equivalence of simple parallel and series circuits. The process that has just been described is the process of designing an L section to match two resistances. Since the L section is the building block from which more complicated circuits such as the pi and T are constructed, it is necessary to understand it thoroughly before taking the next step.

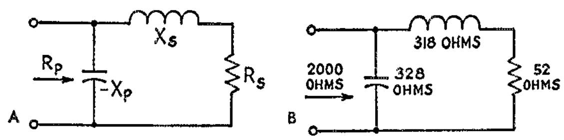

Fig. 5 — Typical practical circuit corresponding with Fig. 4.

Summary of L Section Design

At this point it is well to summarize the step-by-step process of L-section design because the same procedure is used in any network calculation, whether it is the simple L section or a more complicated type:

1) Given the two resistance values to be matched, place the smaller in the series arm of the circuit (R_S) and the larger in the parallel arm (R_P).

2) Find the ratio R_P/R_S.

3) From Equation 5 — or, when Q is 10 or more, from the simpler form of Equation 6 — find the required Q for matching. Alternatively, use Fig. 3 to find Q.

4) From Equation 3B find X_S. X_S may be either inductive or capacitive. The choice will depend on the purpose for which the circuit is to be used, as discussed below.

5) From Equation 2B find X_P. The reactance used in the actual circuit will be of the opposite type to that chosen for X_S.

These five steps determine all the necessary values, but one more is necessary for arriving at circuit constants:

6) Convert the reactances to inductance and capacitance. The following formulas may be used:

$$L = 0.159\frac{X}{f} \qquad (7)$$

$$C = \frac{159{,}000}{fX} \qquad (8)$$

where L = inductance in μh.
C = capacitance in $\mu\mu$f.
X = reactance in ohms
f = frequency in megacycles

Choosing the Kind of Reactance

Purely from the standpoint of matching, either inductive or capacitive reactance can be selected for the series arm and the circuit performance will be exactly the same. The circuit of Fig. 5B could be changed to that of Fig. 6, for example, and the tube would still see a purely-resistive load of 2000 ohms. However, in this particular

[5] This is called a "circulating" current, since it is confined to the loop formed by the inductance and capacitance alone. If there is difficulty in visualizing how a current can exist in such a loop with no current coming from the source of energy, it may help to recall that if the inductance and capacitance were perfect (they never are, of course) any energy supplied to them would be passed back and forth between them, in their electric and magnetic fields, without loss and so a current could circulate in such a circuit forever. Hence no continuous supply of current is required from the source. However, the source does have to supply the energy originally. This transfer from the source to the circuit takes place in an initial "transient" state that is not covered in ordinary circuit theory. The latter assumes "steady-state" conditions — i.e., it deals with what goes on after equilibrium has been reached in the circuit.

application no doubt Fig. 5B would be chosen in preference to Fig. 6, for the reason that harmonic suppression would be better with the former circuit. In Fig. 5B harmonics generated by the tube tend to be by-passed through the shunt capacitance, and are choked off from the 52-ohm load by the series inductance. In Fig. 6, they would be more or less forced to flow to the load

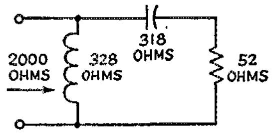

Fig. 6 — An alternative form giving the same impedance ratio.

because the inductance does not by-pass them effectively and the capacitance transfers them readily to the load.

In short, the choice frequently is determined by considerations that have nothing to do with impedance matching as such. In each problem, such things as harmonic suppression, d.c. feed, whether one terminal of a condenser may be grounded or whether both terminals must be insulated from ground, and similar points not related to matching impedances should be given consideration in arriving at a decision.

There are times when a free choice is not always possible or convenient, as when one of the resistances to be matched has unavoidable reactance of one kind or the other associated with it. This occurs frequently in antenna problems. Some typical cases will be discussed later.

Efficiency

The reactances in the foregoing discussion have tacitly been assumed to be completely loss-free. While this is never so, the power loss in the circuit itself is small, in the average case, and can be neglected as a factor in the circuit design. Such losses as occur are almost entirely in the coils. Air condensers, at least at frequencies below 30 Mc., have extremely low losses.

The power loss in a coil depends upon the inherent Q of the coil — that is, the ratio of coil reactance to coil resistance. (This is not the Q figure used in the calculations described above; the latter is the "circuit" or "operating" Q. See Footnote 4.) In circuits handling appreciable power, the coils are generally of good-enough construction to have Q's of the order of 200 or more. If the coil in the circuit of Fig. 5B has a Q of 200, its effective resistance is X/Q, or 318/200. This is approximately 1.6 ohm. For higher accuracy in designing the circuit the coil resistance should be added to the load resistance to find the actual resistance in the load circuit. In most cases this is an unnecessary refinement because the coil resistance usually will be but a small percentage of the total resistance, and the tuning elements usually can be varied over enough of a range to compensate for even greater discrepancies than are likely to arise from this cause.

The efficiency of the circuit is the ratio of the power consumed in the load to the power put into the circuit by the source. It will be equal to the ratio of the actual load resistance to the total resistance, considering the series arm of the circuit. In the example the efficiency is

$$\frac{52}{52 + 1.6} = 0.97 \text{ or } 97 \text{ per cent}$$

when the coil resistance is included. Other cases might not be so favorable; in general, the efficiency will decrease if the coil Q is decreased and if the circuit Q is increased (increasing R_P/R_S). However, an L section uses the minimum possible circuit Q for matching, and so is inherently the most efficient type of matching circuit.

Strays

"Your Novice Accent," by Keith Williams, W6DTY, which appeared in *QST* for November, 1956, has made such a hit that the Hq. has reproduced it in the form of a small pamphlet and is mailing it to all new licensees. Any others desiring an extra copy may obtain one by sending ARRL a stamped, self-addressed envelope.

A bibliography of material published between 1921 and July, 1956, on single-side-band technology has been compiled by the Department of the Navy and released through the Office of Technical Services in the Commerce Department. Entitled "Single Side Band in Communications Systems," the bibliography (PB111837) may be obtained for $2.75 a copy from the Office of Technical Service, U. S. Department of Commerce, Washington 25, D. C.

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