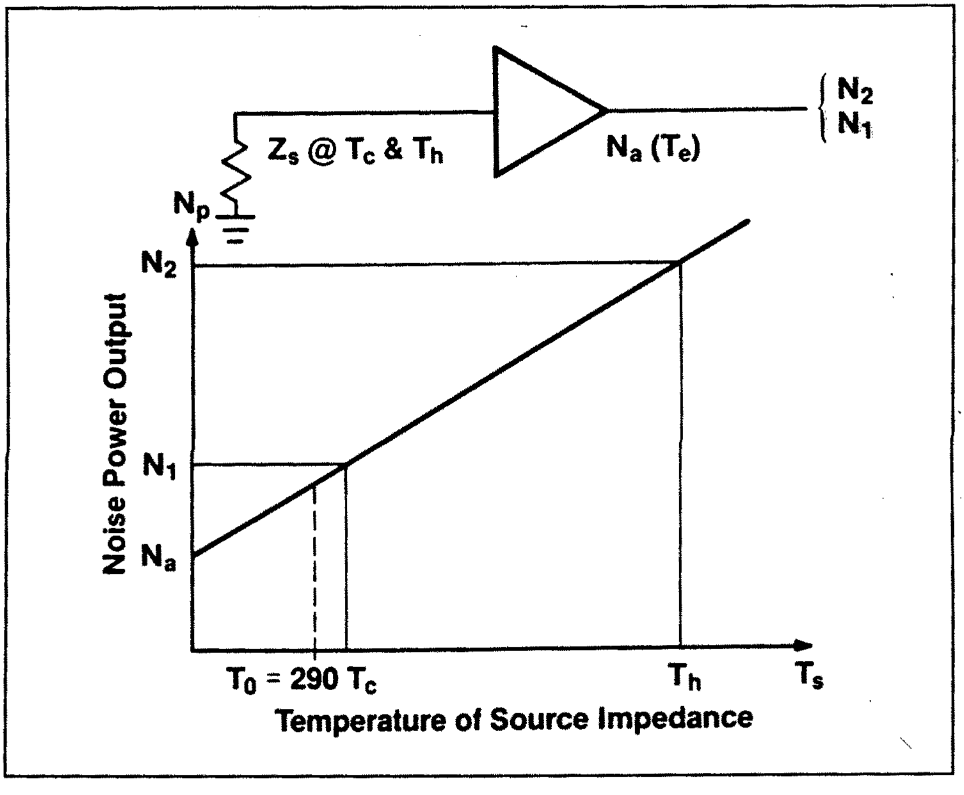
Noise Figure Measurements
by
Al Ward
WB5LUA

The Hewlett Packard HP 8970 Noise Figure meter has been very popular at most of the noise figure measuring events. With its' digital readout to 0.01 dB, how could you ask for anything more accurate? The 0.01 dB resolution is great but as with any piece of test equipment there are limitations in its accuracy. Hewlett Packard talks of a +/- 0.2 dB accuracy which to a large degree is operator controllable. I first published a paper on this subject in the 1988 Central States VHF Society proceedings. This later paper provides greater discussion on the subject.

This paper will start with a quick review of the noise figure measurement and then begin discussion of the items that can effect measurement uncertainty and how to reduce them.

The Noise Figure Measurement by the Y-Factor Method



Measuring noise figure is accomplished by injecting two levels of noise at the input to the DUT and measuring the resultant output noise levels and then performing some math. This is termed the Y Factor method where

$$Y = \frac{N2}{N1} = \frac{kGB\,(Th + Te)}{kGB\,(Tc + Te)}$$

where Th = Noise source temperature when "on"
Tc = Noise source temperature when "off"
Te = DUT noise temperature (Kelvin)
k= Boltzmann's Constant= 1.38E-23 Joule/K
G=Gain expressed as a ratio
B=Bandwidth (Hz)

rewriting the equation

$$Te = \frac{Th - YTc}{Y - 1}$$

or since $NF = \frac{Te + To}{To}$

$$NF = \frac{((Th/To)-1) - Y\ ((Tc/To)-1)}{Y - 1}$$

If Tc = To = 290 Kelvin then

$$NF = \frac{(Th/To) - 1}{Y - 1}$$

Converting to Noise Figure in dB yields

$$NF\,(dB) = 10\ LOG\ ((Th/To) - 1) - 10\ LOG\ (Y - 1)$$

The term 10 LOG ((Th/To) - 1) is also the Excess Noise Ratio (ENR) produced by the noise source

Therefore:

$$NF(dB) = ENR - 10\ LOG\ (Y - 1)$$

where Y is the output signal to noise ratio when the noise source is producing it's ENR at the input to the DUT

Example
ENR = 5.2 dB and
Y = 6 dB or 4 expressed as a ratio
therefore:

$$\begin{aligned} NF(dB) &= 5.2\ dB - 10\ LOG\ (4\text{-}1) \\ &= 5.2\ dB - 4.77\ dB \\ &= .43\ dB \end{aligned}$$

Gain is calculated from the slope of the noise power versus noise source temperature curve.

$$G = \text{Slope} = \frac{N2 - N1}{Th - Tc}$$

Factoring out the effect of the second stage noise figure yields the noise figure of the DUT by itself

$$NF = NF\ (\text{total}) - \frac{NF\ (\text{second stage}) - 1}{\text{Gain (DUT)}}$$

Important Specifications for Noise Sources

The HP 346A noise source has a nominal 5.2 dB ENR while the HP346B has a nominal 15.2 dB ENR. Generally the higher ENR noise source is used to measure very high noise figure devices such as mixers, etc. With low noise amplifiers, the lower ENR necessitates having a receiver, i.e. the noise figure meter, with a lower dynamic range, thereby making the measurement more accurate. The RSS uncertainty of the ENR for both noise sources is

+/- .09 dB from 10 MHz to 12 GHz

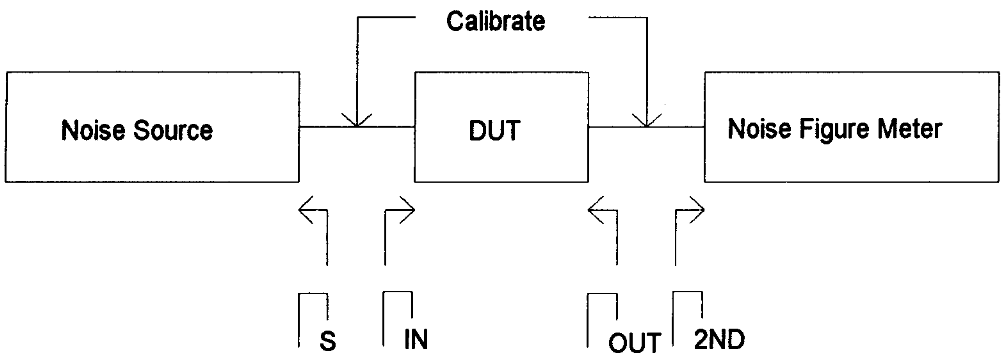
and +/- .11 dB from 12 GHz to 18 GHz.

The maximum VSWR or reflection coefficient for both noise sources is as follows:

10 - 30 MHz	30 - 5000 MHz	5 - 18 GHz
1.3:1 (0.13)	1.15:1 (0.07)	1.25:1 (0.11)

The real difference between the two noise sources comes into play when analyzing the change in VSWR or reflection coefficient with the noise source "on" and "off". With the HP346B, the change in reflection coefficient is not specified but is typically 0.02 to 0.035. With the HP346A, the change is specified at 0.01 maximum. Having a noise source with a constant impedance as it is being gated "on" and "off" by the noise figure meter improves the accuracy of the noise figure measurement and also provides increased confidence that the amplifier or DUT is tuned properly for lowest noise figure into 50 Ω.

Effect of VSWR on Noise Figure Measurement



The HP 8970 noise figure meter is calibrated by bypassing the device under test (DUT) which happens to the be the low noise preamplifier (LNA) or converter and measuring the second stage noise figure. The second stage noise figure is really the noise figure of the HP 8970 noise figure meter. Both noise figure and gain are then measured after the DUT is substituted into the system. The technique of substituting in the DUT uses the "available gain" technique whereby the result takes into account the mismatch that occurs at the input and output of the DUT.

As an example, when the HP 346A noise source is pulsed on, the noise being generated by the noise source is now T_{hot} which is roughly 5.2 dB higher than the noise source would produce if it were not pulsed on. The noise source is now connected to the DUT. If the DUT input VSWR is 1.0:1 then all of the noise power is dissipated in the DUT. If the DUT input VSWR is not perfect then some of the noise power is reflected back to the noise source. If the noise source has a 1.0:1 VSWR then all of the reflected noise power is dissipated or absorbed in the noise source. If, however, the noise source is not perfect then the residual reflected power is re-reflected and the cycle repeats.

The measurement uncertainty is effected by both the source and load VSWRS and how they interact with each other. Depending on the phase of the VSWRs they may add in phase or out of phase, producing either more gain or less gain. If the exact phase of the reflected wave is known, then the actual uncertainty can be calculated. Without measuring the phase of the reflected wave, only a window of uncertainty can be calculated. The worst case uncertainty is the mismatch loss (ML) where

$$ML = 20 \log (1 \pm \Gamma_1 \times \Gamma_2)$$

$$\text{where } \Gamma_x = \frac{VSWR_x - 1}{VSWR_x + 1}$$

Example	Γ s =	.07	(1.15:1 VSWR)
	Γ in =	.33	(2. 0:1 VSWR)
	Γ out =	.20	(1. 5:1 VSWR)
	Γ 2nd =	.33	(2. 0:1 VSWR)

ML (input) = +.198 dB
-.202 dB ~ +/- .2 dB

ML (output) = +.555 dB
-.593 dB ~ +/- .57 dB

RSSing suggests that the gain uncertainty be +/- .6 dB. OK but more important is the fact that the gain measurement also affects the noise figure measurement since the DUT gain is used in factoring out the effect of the second stage noise figure. However, most of the noise figure uncertainty is due to the input mismatch loss uncertainty. Several techniques can be used to minimize errors. They include using isolators on the input to the DUT and input to the second stage which in the case of measuring a preamplifier is the noise figure meter. An alternative is to use an attenuator on the input to the second stage but as we will see later, this increases the noise figure of the second stage which also increases noise figure uncertainty. Some compromise is necessary. The best solution is to design the DUT with very low input and output VSWR. Good output VSWR is relatively easy compared to designing for a low input VSWR coincident with low noise figure.

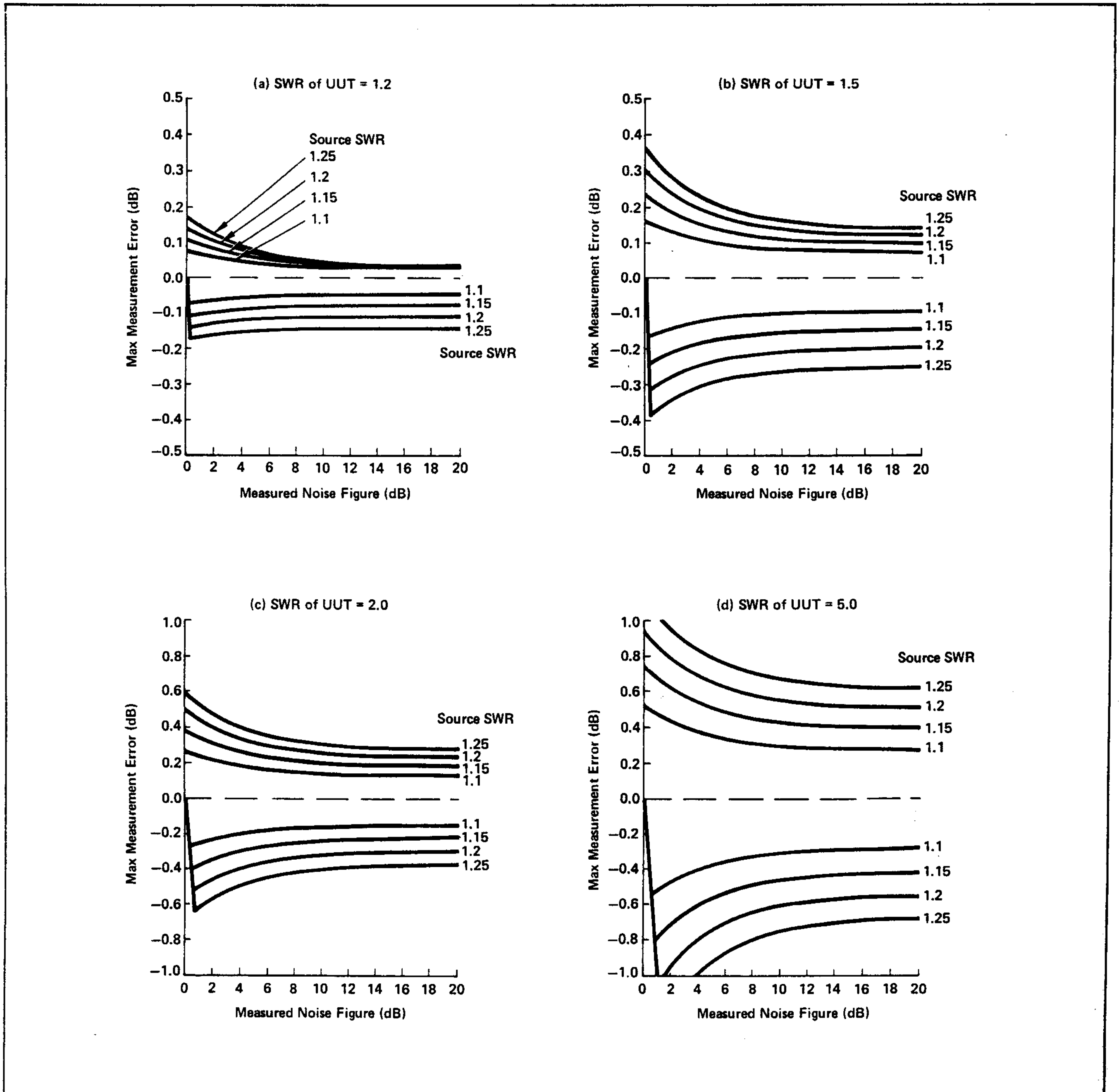
Above all !!!!!! Use a noise source with as low a VSWR as possible to minimize the re-reflection of noise energy back into the DUT. If the noise source VSWR is 1.0:1 then the effect of a high input VSWR of the DUT is unimportant! The end result is the noise figure measurement made on a high input VSWR DUT will still be very accurate.

The following table shows the effect of noise source and DUT input VSWR on mismatch loss which directly relates to noise figure uncertainty - because generally phase is unknown. The equivalent reflection coefficients are also listed.

Mismatch Loss	Noise Source VSWR	DUT input VSWR
+\- .19 dB	1.05:1 (.024)	17.0:1 (.89)
+\- .11 dB	1.05:1 (.024)	3.0:1 (.50)
+\- .07 dB	1.05:1 (.024)	2.0:1 (.33)
+\- .81 dB	1.23:1 (.100)	17.0:1 (.89)
+\- .45 dB	1.23:1 (.100)	3.0:1 (.50)
+\- .29 dB	1.23:1 (.100)	2.0:1 (.33)

An input VSWR of 17.0:1, which correlates to an input return loss of 1dB, was chosen since it was actually measured on a low noise 432 MHz preamplifier of standard design, which by the way was in use on 432 EME at my location in 1975!! The table clearly shows that if a low VSWR noise source is used that the uncertainty in measuring a high input VSWR preamplifier is reduced to an acceptable value. How about when you measured that low noise amplifier on the old home-brew 5722 gas discharge noise source that had a 1.2:1 VSWR! I used to think it was good enough. It probably was the best we could do at the time.

The graphs that follow show the worst case errors in the measured noise figure caused by rereflections between the noise source and DUT



From Hewlett Packard Application Notes 64-3

Effect of Source Mismatch on Actual Noise Figure of DUT

Problem: How sensitive is the noise figure of an amplifier to a change in source impedance?

Discussion: This depends on the type of device and the topology of the input matching network. As the impedance of the noise source changes between the Th (on state) and Tc (off state), the amplifier noise figure and gain may vary, causing measurement errors. Having a low VSWR noise source with minimal change in VSWR between the "on" and "off" states will provide the most accurate measurement of the amplifier.

Another variation of the problem occurs when the amplifier is placed at the antenna. What is the VSWR of the antenna? How much different is it as compared to the noise source? At frequencies below 1 GHz the VSWR of the noise source is less than 1.05:1 Is your antenna that good? I know mine isn't. Most likely it is around a 1.2:1 or even worse. What effect does the different source impedance presented by the antenna have on the amplifier performance? This subject has been studied by many for years and is very complex to analyze properly.

This problem is really a two part problem. First when the amplifier is tested on the noise figure meter with a low VSWR noise source the amplifier was really optimized for a near perfect 50 ohm load, with a VSWR of 1.05:1 as an example. In the actual system, the amplifier may be presented with an antenna impedance not as close to 50 Ω as was the noise source. As an example, let's say the antenna has a 1.23:1 VSWR which is a fairly good match by most standards. The corresponding return loss is 20 dB. Not to bad. Since the amplifier was optimized on the near perfect system, there is the potential for an increase in mismatch loss or an increase in noise figure when the preamplifier is attached to the antenna. Like the uncertainty terms discussed earlier, if the phase of the reflections are known then the exact effect on noise figure can be calculated. Since all we know, generally, is the VSWR, we can calculate the magnitude of the uncertainty associated with the higher antenna VSWR. Using the numbers presented in the last table for an LNA with a input VSWR of 2.0:1 suggests a +/-.07 dB uncertainty for a 1.05:1 VSWR source versus a mismatch of +/- .29 dB for the 1.23:1 VSWR of our antenna. Taking the difference yields a potential uncertainty of .29 dB - .07 dB = 0.22 dB. This way may not seem like much, but it indicates that the system noise figure could have increased by as much as 0.22 dB. In an EME system at 432 MHz with a 0.4 dB measured noise figure, the increase to 0.62 dB can correlate to a loss of system sensitivity of 1.2 dB.

A change in system sensitivity is calculated by taking the ratio of the two system temperatures (k). The noise figures must be first converted to noise temperature and then substituted into the following equation

$$\text{Sensitivity Change} = 10 \log \frac{Ta + Te_1}{Ta + Te_2}$$

where Ta = Antenna Temperature (Kelvin) typically 25K??
and where Te_x = Receiver Temperature (Kelvin) = To (NF - 1)
remember that NF is a ratio and not dB

For example

$$\text{Sensitivity Change} = 10 \log \frac{25K + 44.5K}{25K + 28K}$$
$$= 1.2 \text{ dB}$$

Even though it may go against Murphy's Rules, the actual system noise figure may be better on the antenna and it could also be relatively unaffected by the change! The solution to the uncertainty is to tune the antenna for as low a VSWR as possible. Shoot for less that 1.1:1 VSWR at the receive port of the T/R relay. Some amateurs have also experimented with tuning the LNAs on the antenna by injecting noise into a separate antenna in the proximity of the array.

The second part of the problem relates to how sensitive the amplifier is to source VSWR changes. With the help of gain /noise figure contour circles, the sensitivity of the device to various impedances is understood. Generally the input matching network is designed to transform the 50 Ω source impedance to the device reflection coefficient that produces minimum noise figure. What happens to the device noise figure when the source impedance varies from 50 Ω? The noise figure will increase but by how much? We already discussed the effect that mismatch will have on noise figure but what about the sensitivity if the amplifier to source impedance changes?

The equation that will predict the change in noise figure for the amplifier is very similar to the formula that generates the noise figure circles for the devices by themselves.

$$F = Fmin + \frac{4\,Rn}{Z_0} \; \frac{|\Gamma_s - \Gamma_0|^2}{|1+\Gamma_0|^2 \bullet (1-|\Gamma_s|^2)}$$

Fmin is the noise figure on the amplifier when Γ_0 is equal to 0.0 (or when the source impedance is Z0 or 50 ohms). Γ_s is the reflection coefficient of the source. Rn for the amplifier, which is the rate of change of noise figure as the impedance varies from 50 ohms, needs to be measured or calculated. Rn will vary depending on the type of matching network chosen.

Although very complex to model, the solution to this equation for a given amplifier design will give insight into how an amplifiers' noise figure varies with source impedance.