Reflections on the Reflection Coefficient: An Intuitive Examination

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Much of modern electronics is steeped in mathematics, some of it very complicated and sometimes difficult. The math is powerful, a fact beyond dispute. The strength is in the generality afforded by a mathematical description; one equation can describe an entire family of complicated phenomena.

It is easy to become entrenched in the mathematics of a subject. The utility and even the very structure can be exciting. However, an overly analytic approach can cause some things to appear to be more complicated than they really are. This may be the case with the subject of this note where we ponder the nature of the voltage reflection coefficient.

Power Transfer and Impedance Matching

We begin our discussion with a look at a source of energy, shown in Fig 1. This could be an ac generator, such as an RF signal source, or something as simple as a battery. The source is modeled as a voltage source with a magnitude of 2. The generator has a series resistance of value R_0. A typical value for R_0 might be 50 ohms. Our generator is terminated with (attached to) a resistance of value R_X. The "x" indicates that the value might not be known. We assume initially that the impedances are simple resistances; reactance is re-served for a bit later.

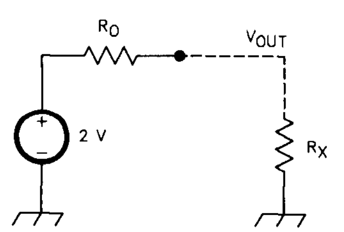

Fig 1

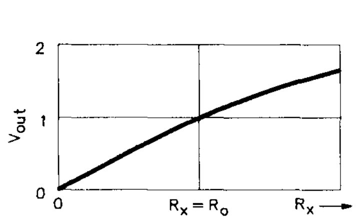

Fig 2

The output voltage is well established. Assume that R_0 is fixed and known, but R_X is allowed to vary. The output voltage is found with Ohm's Law. Some results are plotted in Fig 2. When the termination resistor, R_X, is a short circuit, the output is zero. A voltage appears when R_X is nonzero, and the output continues to climb as R_X gets larger. It does not climb forever; when R_X has become extremely large, V_{out} reaches 2 volts.

A corollary result relates to power. The power delivered to the load, R_X, is a function of the load resistance. The calculation yields the familiar result that the power is maximum when $R_X=R_0$. Either larger or smaller values of R_X will cause less power to be dissipated in R_X. The load is said to be matched to the source. The power appearing in the matched load is termed the available power from the generator.

Further Reflections

Fig 3 shows a modified version of the earlier signal source. A second voltage generator has been added. The second generator, with a 1-volt strength, floats below the earlier V_{out} node, with an output labeled with the Greek capital letter gamma (Γ). The polarity of the added generator is opposite that of the first generator. The voltage at gamma is 1 volt less then V_{out}.

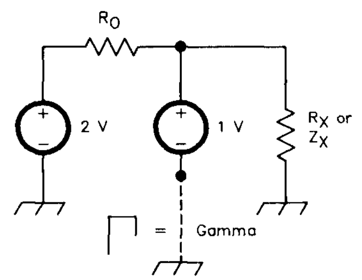

Fig 3

The value for gamma (a voltage) is interesting and useful. Assume that a very high impedance is attached to the R_X port of Fig 3. Little current flows, so the output voltage (V_{out} of Fig 1) is 2, the open circuit value. The corresponding value for gamma is 1 volt.

If the terminating impedance is set to R_0, the reference, or source resistance, the signal across the termination is 1 volt, causing gamma to become zero.

The third case of interest is a short-circuit termination. This causes gamma to be –1. If the experiment

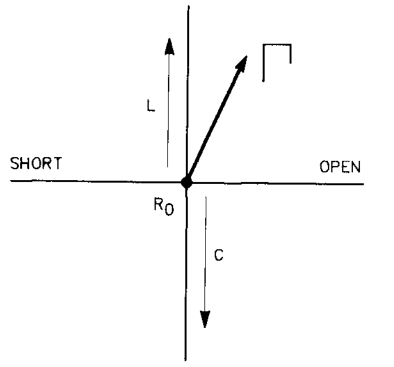

Fig 4

described was done with batteries, a negative voltage would happen for gamma. If the generators were ac sources, the short-circuit termination would yield a gamma with magnitude of 1. It would, however, have a phase that is 180 degrees away from the driving 2-volt generator.

Gamma has significance. Gamma was 0 volts when the perfect impedance match was present, $R_X = R_0$, but was highest when there was a severe impedance mismatch. Gamma is, hence, a measure of the degree of mismatch. Energy that is not transferred from a source to a termination is said to be reflected. Accordingly, gamma is called the voltage reflection coefficient.

Our examples have used pure resistance for the termination. The general case would allow complex terminations: impedances consisting of both resistance and reactance. The reactance would arise from either inductors or capacitors. Fig 4 is a plot showing a gamma value resulting from a complex termination.

Consider first the resistances we have already discussed. Gamma takes on values from –1 to 1 for terminating resistors, R_X, from a short to an open circuit. This variation is shown on the horizontal axis of Fig 4. This line, of length 2, is a "map." All possible resistances from 0 to very large values correspond to points on the horizontal line. The most exciting detail of this result is that a strictly infinite range of resistance has been mapped onto a line of finite extent.

Adding reactance to a resistance generates impedances that map to points that are no longer on the horizontal axis. An inductance with the resistance generates a reflection coefficient point somewhere above the horizontal line. Resistor-capacitor circuits generate points below the line. Recall that a complex impedance is represented by a complex number, one with a "real" and an "imaginary" part. It is reasonable that a complex impedance would generate a complex reflection coefficient. Fig 4 is just a polar plot of various complex reflection coefficients.

Just as resistance was allowed to vary over a very large range, reactance can vary from large capacitive (negative) values to zero to large inductive (positive) values. Rather, the behavior is more complicated. What is found is that any possible impedance, Z=R+jX, will have a corresponding gamma value that has a magnitude of 1 or less. Every possible complex impedance with a positive value of R will generate a unique point within a circle of unity radius.

The reader with a mathematical bent can analyze the circuit of Fig 3 with a complex termination, Z, and show that

$$\text{Gamma} = \frac{Z - R_0}{Z + R_0}$$

A common variation of this equation divides both the numerator and the denominator by R_0 to "normalize" the result, producing

$$\text{Gamma} = \frac{z - 1}{z + 1}$$

where the lower case value is the normalized impedance, $z = Z/R_0$.

Circles

Fig 5A summarizes the polar plot of reflection coefficients. Associated with a given gamma is a point. A vector from the origin to that point has a length equal to the magnitude of gamma. The angle between the vector and the horizontal line is the angle of the reflection coefficient. The various circles shown in Fig 5A represent gamma values of 0.2, 0.4, 0.6, 0.8 and 1.

A variation of the polar plot of reflection coefficients is presented in Fig 5B. The circles of constant gamma have been eliminated. Instead, they have been replaced

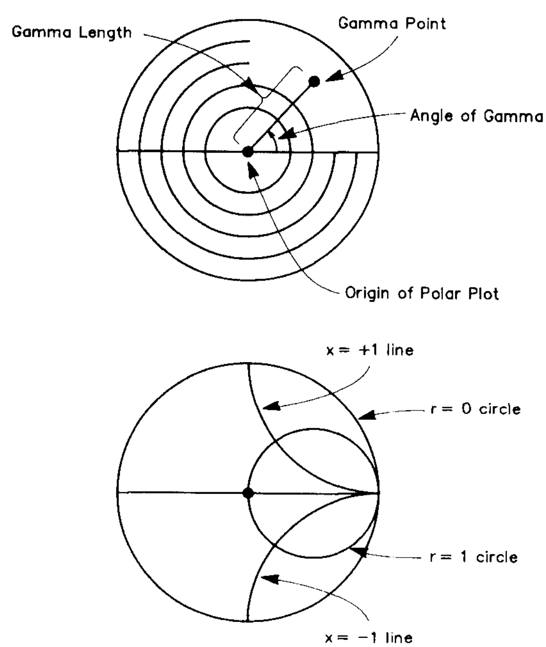

Fig 5A (top), 5B (bottom)

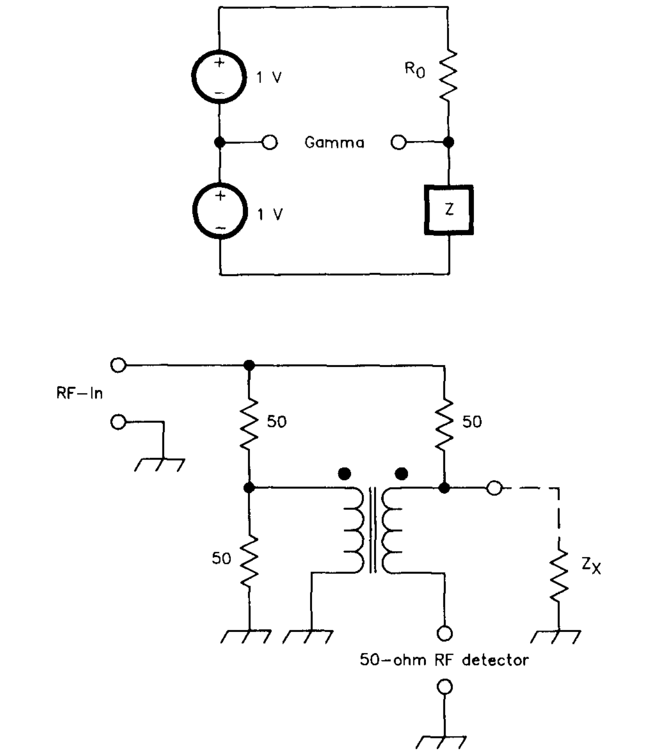

Fig 6A (top), 6B (bottom)

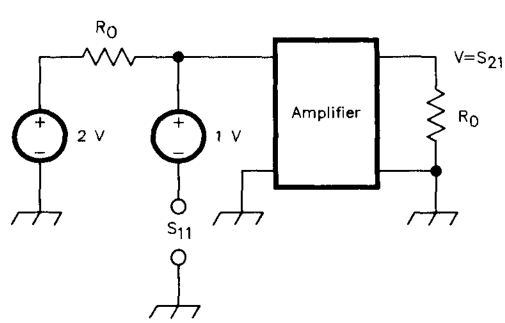

Fig 7

with smaller circles or circular segments. These represent lines of constant resistance or reactance. The horizontal line is our earlier map of all possible resistances; this is a line of impedances with zero reactance. A circle is shown that passes through the origin. This is a collection of impedances that all have constant resistance of R_0, or normalized r=1, at all possible reactance values. Other circles (missing from Fig 5B) are positioned inside the "unit circle" and represent other contours of constant R; all of these pass through the right-hand end of the center line. Two circular segments are shown. These represent lines of constant reactance with normalized values x=1 and x=-1. Even the outside edge is significant. The unit circle is the map of all possible impedances with zero resistance. The upper half of the unit circle represents all possible values of inductive reactance while the lower half maps all possible capacitive reactance.

Fig 5B is, of course, the familiar Smith Chart. This design and analysis tool is usually associated with transmission lines. Clearly, that is not required. The Smith Chart is useful as a generalized view of the impedance world. It is most useful for the study of transmission lines that have the same characteristic impedance as the R_0 value of the chart. A mismatched (or matched) transmission line has a constant reflection coefficient along the line. The angle of gamma is related directly to the length of the line, or the position along the line. This is the basis for the usual circular plots that are done on the Smith Chart.

Further Extensions

The figures presented so far have illustrated a way to view the famous Smith Chart. The circuit forms are most convenient as a match to standard mathematics. Shown in Fig 6 is a variation that comes closer to a familiar measurement. Fig 6A has the previous 2-volt source replaced with a pair of 1-volt sources. The 1-volt subtraction that we did with a floating generator in Fig 3 is now done by measuring the voltage difference between the V-out node and the tap between the two generators.

Fig 6B shows a practical circuit, the familiar return-loss bridge. The two left-hand resistors serve to generate a reference at their junction. The current balun allows a grounded RF detector to "appear" between the floating nodes where gamma is measured. Other bridges can be similarly viewed.

Fig 7 is another extrapolation of the same ideas. The gamma measurement tool of Fig 3 is used to excite an amplifier or other network of interest. The amplifier output is terminated in a pure impedance, usually the same R_0 that was used for the input "bridge." The resulting circuit forms a definition for scattering parameters, the bread-and-butter tool of the RF engineer. The so-called forward gain results when the measured output is compared with the 1 volt that would appear across a matched load at the input generator. The amplifier output could be driven with a gamma measuring set, or network analyzer, while simultaneously terminating the input in R_0. The resulting reverse signals would then be s_{22} and s_{12}.

Scattering parameters are usually defined in terms of waves. While the more detailed definitions are extremely powerful, they often obscure the simplicity (yes, simplicity!) afforded by the scattering parameter view of a circuit.

Improved Mechanical Design for the Helical Antenna

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The helical antenna is an effective way to obtain circular polarization, but the wooden mechanical design of the antenna as typically used by amateurs leaves something to be desired. In this article, an almost all-metal mechanical structure is presented. It can be made from readily available material and can be built with common hand tools.

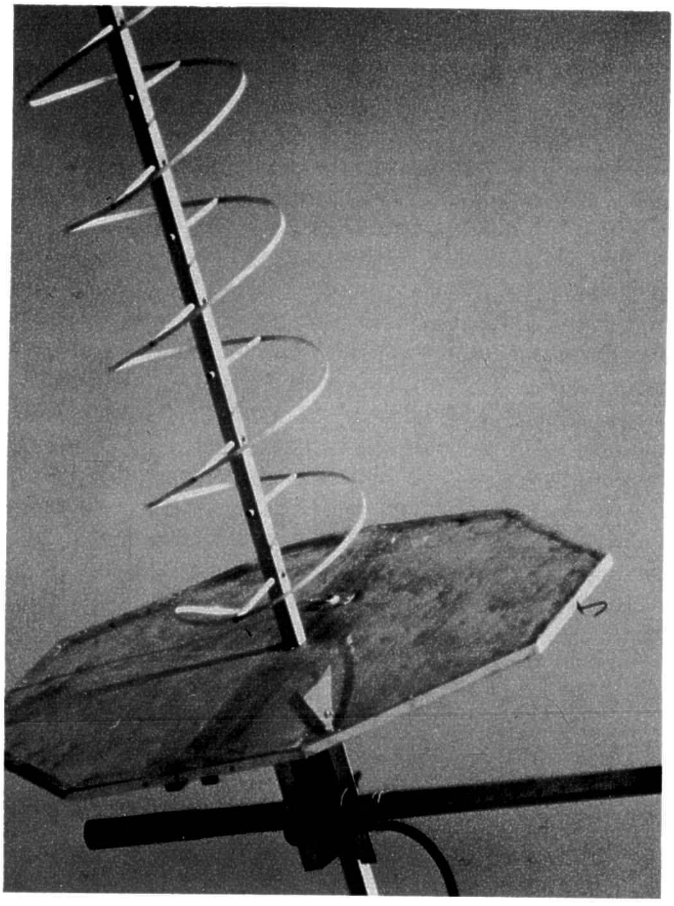
The helical antenna is a widely used satellite antenna in the amateur community as well as in the professional world. It is an antenna that can be built easily and provides good performance since the broadband nature is very forgiving of dimensional errors.

Past Techniques

For amateur use, the only shortcoming of the helical antenna is the mechanical construction. I've spent hours in local lumber yards trying to find the exact piece of wood for the antenna element supports.[1] That is, a piece of wood without knots and perfectly straight. Once the perfect piece of wood is found and brought home, antenna construction is well underway. Protection of the wood from the kind of weather it will experience on the top of the tower is an art in itself. Probably the best way to protect the wood is to apply several coats of a good marine varnish or paint (in a favorite color). The proper selection of paint or varnish will allow the antenna to last through several good years of operation. However, even with the best paint, the antenna will require repainting every few years.

A hidden defect of the wooden-support approach does not appear for several months, perhaps a year or so. Most wood that can be purchased at local lumber yards has not been thoroughly dried before being cut to the shape or size the lumber yard sells. So in most cases, the wonderful antenna begins to twist, sag or bend after a time as the wood really dries out. Not only does the antenna begin to look bad, but the bend or twist changes the pattern thereby losing the gain and circularity which was the reason for building this type of antenna in the first place.

Okay, so wood is not the best material for the antenna even though it's readily available. Another recommended material for the support is fiberglass. Up until recently, (at least in this area) fiberglass tubing was not easily found. Paint roller extension handles are now available in approximately 4-foot lengths at local paint and discount houses, but information is not available as to the weatherability of this material. It is lighter than wood, but equal to or heavier than aluminum of the same size. Splicing for the desired length and strength can be a problem as well. It is definitely more expensive than wood or aluminum.



Helical Front (photos courtesy KØRL)

[1]Notes appear on page 16